WHY DO YOUR FINGERS ALTERNATE THEN MEET AT THE CENTER OF MASS OF THE ROD? AND WHY IS THE PROCESS NOT REVERSIBLE?

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ABSTRACT

This paper is made of a letter that I e-mailed to my students at the end of the course described in my paper *Logic and Inequalities* [†]. In one of the lectures on inequalities, I showed to my students the well known trick with a rod on two fingers and promised to explain why it is all about inequalities. The text of my letter was mostly borrowed from John Rennie's answer to a question on Physics Stack Exchange [‡] Since it is used in teaching, I hope that it is fair use. I added some additional discussion, and a question:

Why is dynamic friction less than static friction?

My letter to students

Dear Students,

one of you asked a question:

> *Can you explain why the process is not reversible when finding the centre of the mass? The question you raised in the lecture was interesting but I can't find the answer if you gave one.*

The question refers to a trick that I showed in one of my lectures; it is widely described on the Internet, and I copy the description of the experiment and a diagram from this post on the Internet [§] written by John Rennie.

> *Hold a pen (or pencil, ruler etc.) using your two index fingers with your fingers at the ends of the object. Now move your two*

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[†] A. Borovik, *Logic and Inequalities: A remedial course bridging GCSE and undergraduate mathematics*, Selected Passages From Correspondence With Friends, 3 no. 1 (2015), 1–6; bit.ly/1FZDj0P.

[‡] John Rennie, Answer to *Why do your fingers alternate then meet at the center of mass of the object?*

[§] Ibid.

fingers toward each other. Assuming the frictional force pro-
vided by both fingers is identical, they should meet at the cen-
ter of mass of the object, with your fingers alternating between
being stationary and moving. When going backwards from the
centre, one finger stays stationary the whole time. Why is this
the case?

There is a long discussion in the post, but I prefer a short 'hand-wavey'
explanation also present there:

*the **closer** to the centre of mass a finger is, the **greater** the reac-*
tion force and thus the frictional force exerted, the finger with
***less** friction moves until it is holding **more** weight and the fric-*
*tional force becomes **too large** ... [Emphasis is mine – AB.]*

As you can see, it is all about inequalities.

Alternatively you may wish to read a much more detailed answer (which
also uses inequalities) given by John Rennie [†] and accompanied by nice
diagrams – decide for yourself which way is preferable for you:

John Rennie explains:

If we take a side view the initial state of your system looks like
this:

mass *m*, length *l*

mg/2   mg/2
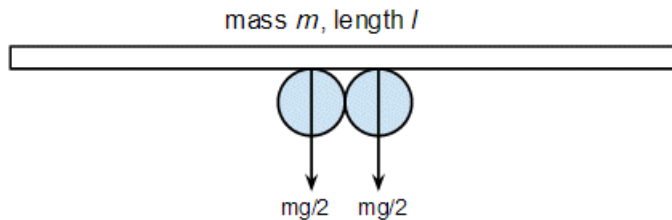
It should be obvious that the force at each finger/ruler contact
is $mg/2$, and therefore the frictional force is:

$$F = \frac{1}{2}mg\mu_s$$

where μ_s is the coefficient of *static* friction. Now you start to
pull your fingers apart, and one of your fingers will slide first.
Which finger will slide isn't predictable because it depends on
minor details like how sweaty the two fingers are, whether you

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have minor muscle tremors, whether the fingers are at exactly the same angle and no doubt many other details. Let's assume the right finger slides first so the system now looks like this (I've exaggerated the amount of movement to make the diagram clearer):

centre of rod

F$_1$ F$_2$

d$_1$ d$_2$

mg
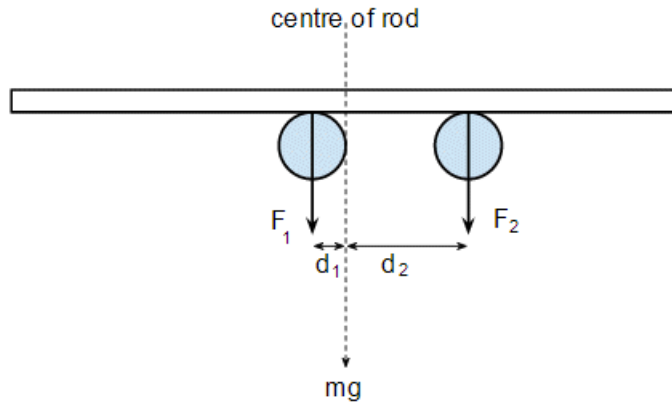
I've labelled the finger that didn't slide 1 and the finger that did slide 2. The forces are no longer the same because the system is no longer symmetrical, though of course they must still add up to mg.

Now, the frictional force at finger 1 is $F_1\mu_s$, and the frictional force at finger 2 is $F_2\mu_d$, where μ_d is the dynamic friction. Note that dynamic friction is generally lower than static friction so $\mu_s > \mu_d$, though actually this won't make a difference to our conclusions. So we have three possibilities:

(1) $F_1\mu_s > F_2\mu_d$ in which case finger 2 carries on sliding
(2) $F_1\mu_s < F_2\mu_d$ in which case finger 2 stops sliding and finger 1 starts sliding
(3) $F_1\mu_s = F_2\mu_d$ in which case both fingers slide

It's pretty easy to show that case 1 applies. There are three forces acting, F_1, F_2 and the weight of the rod mg. We'll take moments about the centre of the rod, which means that the moment due to the weight of the rod is zero, and we get:

$$F_1 d_1 = F_2 d_2$$

or:

$$F_1 = F_2 \frac{d_2}{d_1}$$

and because $d_2 > d_1$ the ratio $\dfrac{d_2}{d_1} > 1$ and therefore $F_1 > F_2$.

If we take our condition 1 above and rearrange it slightly to:

$$F_1 > F_2 \frac{\mu_d}{\mu_s}.$$

We just proved that $F_1 > F_2$, and we know that $\dfrac{\mu_d}{\mu_s} < 1$ **because dynamic friction is less than static friction** [emphasis is mine – AB], so we've proved that:

$$F_1 \mu_s > F_2 \mu_d$$

and condition 1 applies.

Our conclusion is that as soon as a finger slips even a little bit the friction between that finger and the rod is reduced while the friction between the other finger and the rod is increased. So whichever finger slides first will carry on sliding.

This is a nice explanation, but it immediately leads to the next question: look at the highlighted sentence above:

Why is dynamic friction less than static friction?

I know a feasible explanation, and it is again all about inequalities.

Discussion

This is the lesson of this simple example:

Subtler effects of the so-called real life are described by inequalities, not by equations.

A natural question arises: why in this case mathematics as taught in secondary school focuses almost entirely on equations?

The answer is simple: because this is **simpler** and therefore **cheaper** to teach. As you can see, it is again explained by inequalities: cheaper.

Festive Greetings and Happy New Year!

I wish you not just luck, but a calm and confident success in all your exams –

Alexandre Borovik

15 December 2018

Disclaimer

The views expressed do not necessarily represent the position of my employer or any other person, organisation, or institution.

About me

I am a research mathematician but I have 40+ years of teaching experience at secondary school and university level in four different countries with four different education systems; since 1998 I am a Professor of Pure Mathematics at the University of Manchester.

I also have an interest in cognitive aspects of mathematical practice; see my book *Mathematics under the Microscope* [†], which explains a mathematician's outlook at psycho-physiological and cognitive issues in mathematics and mathematics education, and touches on many issues raised in this paper. Some of my papers on mathematics education can be found in my personal online journal/blog *Selected Passages From Correspondence With Friends* [‡].

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†A. V. Borovik, *Mathematics under the Microscope: Notes on Cognitive Aspects of Mathematical Practice*. Amer. Math. Soc., Providence, RI. 317 pp. ISBN-10: 0-8218-4761-9. ISBN-13: 978-0-8218-4761-9. Available from the AMS.

‡Selected Passages From Correspondence With Friends. ISSN 2054–7145.