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"FREE MATHS SCHOOLS": SOME INTERNATIONAL PARALLELS

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We teach criteria.

Apocryphal: a President of Harvard University, responding to a question on what was so special about Harvard to justify the extortionate fees.

Abstract

The idea of "free maths schools" run by universities (the media have already dubbed them "elite maths schools", which, in my opinion, is harmful) is relatively new for this country, and some international parallels could be useful. This text provides some personal observations about one of the more extreme examples of "specialist maths schools", my *alma mater*, Novosibirsk FMSh, the preparatory boarding school of Novosibirsk State University in Siberia. In a nutshell,

the School taught criteria.

This paper *is not a recipe for Britain*, but I hope that it could be useful in assessment of projects of "free maths schools".

Disclaimer. Needless to say, all opinions expressed in the present paper are of the author and no-one else. More specifically, the author worked on the present paper in his private capacity; all views expressed here may or may not represent the position of the London Mathematical Society which does not bear any responsibility for the content of this paper.

FMSh: an extreme case of a specialist maths school

FMSh's full name was *Physics and Mathematics Boarding School of the Novosibirsk State University*[†]. It had about 600 students aged

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2010 *Mathematics Subject Classification* 97C50 (primary), 97D20 (secondary).

[†]Website of its current incarnation: `http://nsesc.ru/`.

15–17, of the last two years in the Russian system of school education. Together with the University, it was situated in Akademgorodok, a campus of a research centre of Academy of Sciences near Novosibirsk. Students for FMSh were selected via a system of competitions in mathematics, physics, chemistry ("olympiads") from the catchment area which covered Siberia, Soviet Far East, Kazakhstan and Soviet Central Asia and had a population of about 40 million. FMSh was not unique; indeed the rest of Russia was divided between boarding schools of Leningrad[‡] and Moscow Universities—the latter is known in the West as "The Kolmogorov School"[§](perhaps it is worth mentioning that in learning from the experience of others, we in England are fortunate in having on hand one time Head of Mathematics of The Kolmogorov School, Professor Alexander Veselov of Loughborough University). There were also famous day schools in Moscow and Leningrad, like, say, Schools nos. 2, 57, 444 in Moscow. Of other former Soviet republics, Armenia and Ukraine had their own specialist mathematics schools of high reputation; I have no information about other republics.[¶]

It was a distinctive feature of Novosibirsk FMSh that at the last stage of the selection process a child was invited to spend a month in the Summer School in Akademgorodok, to check how he/she reacted to an intellectually stimulating environment; those eventually accepted into the boarding school were expected to be self-motivated learners with a tendency for an explosive intellectual growth and ability to learn by absorbtion.

As the reader can see, FMSh was a seriously selective establishment.

One aspect of the assessment of potential candidates would perhaps be surprising for a British reader: the previous school record of a child was blatantly ignored; a standard form which had to be filled by a selector during an interview did not even contain fields for school marks. And selectors were, as a rule, professional mathematicians or physicists from research institutes of Akademgorodok, who for that purpose traveled all over Siberia and Central Asia to run regional olympiads and interview successful competitors.

Another interesting detail also goes against the current British cus-

[‡]Physics and mathemtics School no. 45 is now known as Academic Gymnasium of the St. Petersburg University, `http://www.agym.spbu.ru/fizmat.html`.

[§]`http://www.pms.ru/`.

[¶]The website of FMSh, `http://nsesc.ru/about.html`, says that specialist physics and mathematics schools in Moscow, Leningrad, Novosibirsk and Kiev had been founded by the Council of Ministers of the USSR, Decree no. 905 of 23 August 1963.

tom: in the whole system of olympiads, selection interviews, Summer School, FMSh itself the use of words like "gifted", or "talented", or even "mathematically able" was explicitly forbidden. Children in the system were referred to as "interested in mathematics"—and this was more than an euphemism: in conversations with children or in the presence of children, the emphasis was on their *interest* in mathematics and their *motivation* for study of mathematics. (Old habits die hard, and the very words "Gifted and Talented Education" still make me cringe.) Of course, selectors assessed mathematical abilities of an interviewee on several different scales, and these assessments had been recorded—but never discussed with a child. In conversations between selectors, codewords for "mathematically able" were something like "a strong child" or, in more exceptional cases, "an interesting child". Despite a very competitive environment of the School, these conventions were accepted by students; any talk about "abilities" was considered to be in bad taste.

Finally, the word "elite" was simply non-existent. In Britain, the Free Maths Schools have been already dubbed by newspapers and media as "elite" schools[†]; in my opinion, this is a public relations flop with potential to poison the Free Maths School project.

Dr Evgenii Bichenkov and his paper

I was personally involved with FMSh for many years: I was a student there for two years; when I was a student at the Novosibirsk University, I worked part-time in FMSh as a night warden (and had a chance to witness firsthand the psychological dynamics of the student community first from a student's and then from a tutor's viewpoint); for many years, I acted as a selector.

I can tell a lot but I prefer to arrange my reflections on FMSh around much more authoritative testimonies and quote heavily the paper *Physics and Mathematics School in a third of century* written in 1988 by my physics lecturer at the School, Dr Evgenii Bichenkov, one of the key players in the early development of the School. FMSh was a peculiar but very successful establishment, and Bichenkov's paper

[†]See, for example, such publications as *Government To Fund Elite Maths Schools*, `http://news.sky.com/home/politics/article/16118087`, or *Osborne in $600m drive to set up elite maths schools run by businesses and charities*, `http://www.dailymail.co.uk/news/article-2066377/Osborne-600m-drive-set-elite-maths-schools-run-businesses-charities.html#ixzz1jPiYAOMj`.

is interesting because it summarises its founding principles; I hope it could find some use in the discussion of the Free Maths School project.

However a word of warning is necessary: Bichenkov wrote in a different historical epoch, in a different socio-economic, political and cultural environment. His recommendations are not directly applicable in modern day Britain. In particular, his paper reads as a kind of a *manifesto of meritocratic elitism* in education which, in the current educational environment in this country, would need to be handled with care and would require a very measured approach.

What follows are quotes from Eugenii Bichenkov with my comments.

Bichenkov starts by asking two questions:

> What new was brought by the School into the practice of school education? What are the principal results of its activities in teaching basics of sciences at a school level?[†]

Selection of students

Bichenkov answers these questions by first discussing the selection process for the School's students.

> So, what has been achieved by selection of students? I am deeply convinced that the very fact of selection and creation of this collection of children on the basis of selection is beneficial for a child. When they come from their schools, where all roles and places have been already assigned and fixed, children start their internal competition for distribution over the scale of their hierarchy of values. They cannot not to do that—such is their nature and their age. It is important that at that age they are offered decent moral and human rules of competition and shown some good examples. It appears that the Novosibirsk FMSh has succeeded in that.

To anyone familiar with British state schools, this may sound strange: open competition between students. At FMSh, students were constantly challenging each other to solve problems, puzzles and conundrums, and the art of practical jokes was developed with considerable sophistication.

[†]Eugenii Bichenkov's text had been translated from Russian by my school friend from FMSh who prefers to be known only as Owl.

> Next. To what degree was selection determined by true abilities? Did the results match the declared aims?
>
> Here I cannot give a definite answer. In many ways selection can still be affected by chance. The selection is obviously influenced by personal aspirations and interests of the child, by family, teachers, friends, acquaintances; the results of olympiads are affected by competitiveness, persistence, level of maturity, after all. And of course, choice manifests the personality of the teacher, the examiner.

This is perhaps the most serious deviation from anything that could be expected in Britain: please notice that Bichenkov accepts as inevitable that different examiners apply different criteria for selection; it does not matter for him whether the criteria are uniform; what matters is that they should be fair and allow the examiner to select the very best candidates.† We need to accept that two different selection systems will always make different judgments "at the margin". So one needs a system that reliably selects those who are well above this margin, and that distinguishes reasonably well at the margin.

Crucially, selectors were professional researchers. I myself was picked at a regional Olympiad by an interviewer who at that time was a mathematics PhD student; in later years I knew him professionally, as a fellow researcher in mathematics.

I have already mentioned another important feature of the selection process: it was based entirely on competitions in physics or mathematics and face-to-face interviews. Previous school performance was totally ignored—no questions were asked about it.

Choice of teachers

Evgenii Bichenkov continues:

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†Alexander Shen, a famous teacher from School no. 57 in Moscow, pointed out that his school was using a very different approach:

> As to the differences with my experience in school 57, our idea was to make entrance decision based not on our estimates of future success or interest, but on the exam results: I had a strong feeling that fair decisions should be based on some objective criteria, and to put children into the situation when their optimal strategy is to simulate some interest etc. is not a good thing. Also making arbitrary decisions is not a moral thing for teachers, too. But this was just the policy of team where I participated (and I have to say that some other people, not having this policy, seem to be more successful in the selection).

> A question arises at that point about choice of a teacher for children selected for the school. From the very beginning we put forward one restriction on the choice of a teacher—a teacher had to be a scientific researcher working in the Siberian Branch of the Academy of Sciences. For all its apparent weakness, this restriction turned out to be a rather refined and correct criterion of selection; it allowed one to sideline some applicants to a post of a teacher at FMSh who had nothing to offer besides their eagerness to get employment at the school, who had no any other objective qualifications for work with gifted[‡] children.

The word "teacher" here means a teacher of mathematics, physics, chemistry, biology. On day one in the School we were told by the Headmaster that our teachers had no teacher training and that we had to be prepared to accept their occasionally eccentric behaviour. Our teachers could be late for classes and otherwise not very organised (or just too busy in their main job), but they were able to solve, on the spot, almost any problem, and answer almost any question. If they occasionally could not, they honestly told us so, but came back next week with a solution. Their best lessons were more often than not improvised on the spot.

Also, I remember my biology teacher heroically staying awake in the class despite not having any sleep for three days—in molecular biology, experiments could be very long and taxing on the researcher; it was a lesson in itself. Crucially, our teachers shared the academic ethos of the professional research community.

Other disciplines (history, literature, etc.) were taught in a more traditional way, by normal school teachers, although very good ones. While retaining the mandatory state curriculum, these disciplines were allocated less time than in normal schools.[†]

> It turned out that, in the environment of Akademgorodok, being a scientist almost perfectly match the requirements for a FMSh teacher, both in technical and pastoral aspects. We live in a very specific community where we meet each other face-to-face and know by work, and have to constantly reckon with that. We were lucky that from the very foundation of Akademgorodok a scientist was judged by his

[‡]This is the only instance of the use of word "gifted" in Bichenkov's paper—and it is used in a discussion of selection of *teachers*. As I said before, words like "gifted" and "talented" were explicitly forbidden in the School.

[†]I remember how my class covered *War and Peace* in two hours, discussing, in a debate which generated more heat than light, just one topic: Tolstoy's philosophy of history.

> or her work, and judged according to high standards. Under such conditions a bad scientist just could not be a teacher at FMSh, and if this happened, this happened only due to some administrator's mistake which was soon corrected.

And Eugenii Bichenkov concludes his discussion of the choice of teachers with an ominous warning:

> *I do not know how to select teachers in settings other than Akademgorodok.*

It is clear that in our own setting we must somehow learn from the spirit (or perhaps the underlying criteria) Bichenkov is highlighting, be prepared to interpret flexibly, and then to monitor the effectiveness of whatever selection criteria are used.

Besides teachers, FMSh also had tutors who were in charge of general discipline, well-being and upbringing of children. A tutor was in charge of a class of about 20–25 students. Tutors were a mixed bunch, and the criteria for their appointment were less clear. When I worked as a night warden, I had to deal with the tutors a lot—some of them were excellent, some awful.

> From our experience, the principal criterion of selection is personal achievement in a previous job: if an applicant is an engineer, he has to have success in his projects and be full of ideas; if a teacher, then an imaginative one and a school's favourite; if a student, then an academically outstanding one, but also with an inventive soul, a good guy. The school workforce should be open to renewal, to constant in-flow of new people. It should attract people of diverse interests and personalities. One may say that selection should use the principle of mutual complementability of people. In the environment of Akademgorodok, this principle emerged naturally. There were several different schools of physics and their representatives began to mix in the physics department of FMSh, cooperating and enriching each other with their knowledge. At first it happened accidentally since the work at school could not be compared with a work at an university neither in pay nor prestige. [...]

The teacher of my mathematics tutorial class was a first year PhD student, but a very bright and successful one—by that time he had already published some nice mathematical results. He, by the way, in-

troduced me into his area of research—group theory—and later became my MSc project supervisor. But I can confirm what Eugenii Bichenkov was saying: our teachers radiated intellectual confidence and success. This is why they were able to pass on to us criteria of success, of a job well done.

Teaching arrangements

Let us listen to Dr Bichenkov once more:

> I have expressed my opinion about two most fundamental questions for the specialized school: "Who is to be taught?" and "Who should teach?". The third question is "What is to be taught?". I will discuss it using physics as an example, but I will take the risk of drawing some general conclusions. In our teaching practice we have worked out some "boundary conditions" which in many respects guide the development of our courses. Within the formal time frame of the so-called teaching plan, the main principles are the following:
> – Short training period, one or two years. We have to admit that the three year programme of study at the boarding school was not a success.

Perhaps, this was because the school was not about teaching a large body of standard material, but about teaching *criteria*; I discuss this later in more detail.

> – Short terms. The autumn term lasts from 1 September to about 10 December; the next two weeks are reserved for midyear tests and examinations, then follow three weeks of winter vacations when children have a much needed rest from the dormitory. The second term lasts from 20 January to 20 May followed by examinations and summer vacation. In addition, there are short holidays in November and May.

Well this was longer than university semesters are in Britain now. But I can testify that long winter breaks were badly needed, indeed; I used to return to the parental home completely exhausted. My vacations were not rest, they were convalescing.

I have seen something similar in Cambridge: relatively short terms of very intensive study, with long periods of recuperation afterwards.

> – Short weeks. For all the intensity of studies, a working week in FMSh is five days.

I have to mention that in Russia at that time school week lasted 6

days. In FMSh we had classes on Saturday (but on less demanding subjects), but Thursday was a "reading day", with no classes scheduled.†

> – Short lecture courses. No lecture course can take more than two hours a week. At present, the total of compulsory lessons does not exceed 32 hours a week. We came to this constraints not straight away and not straight on. Our starting point was Lavrentiev's† somewhat aphoristic quip:
>
> '*A student should have free time to think about what he is actually taught.*'

This message was very explicitly conveyed to students: we were given a lot of free time with a very specific purpose: this was time to think.

Back to criteria

Finally, Bichenkov discusses content of teaching, taking physics as an example; almost everything that he says applies to mathematics as well.

> The content of physics course in FMSh was shaped by a number of very different teachers. They worked in various fields of physics in different Institutes and they were people of different ages. Being limited in time and striving for expression of their own scientific interests, they could sometimes succumb to oversimplification of scien-

†To give some flavour of a "reading day" I have perhaps to mention that I was spending most Thursdays lying in bed and reading English books: it was my own initiative to set aside an English-only day of the week. The books were either mathematical, like *A Survey of Modern Algebra* by Birkhoff and MacLane and *Finite Groups* by Gorenstein (the former was an undergraduate level, the latter a postgraduate level text; they were provided to me by my mathematics teacher who borrowed them from the research library of the Institute of Mathematics), or traditional English literature (where I started with Rudyard Kipling's *Just so Stories*, then moved to Robert Louis Stevenson's short stories like *The Pavilion on the Links*—remarkable for simplicity of language, and eventually reached more challenging Henry Fielding and Laurence Sterne); alas, I am still a better English reader than writer. Meanwhile, my roommate (who is nowadays a professor of plasma physics in one of the leading universities) was mostly reading books on astronomy and busying himself with design of photometric devices for measuring brightness of variable stars.

†Mikhail Lavrentiev (at that time—President of the Siberian Branch of the Academy of Sciences) was a world recognised expert in fluid dynamics and a hardcore engineering mathematician; but he started his mathematical career in the purest of all mathematics, descriptive set theory of the famous school of Luzin, and he was protecting and supporting *fundamental* science with almost religious zeal. At every Summer School, Lavrentiev gave a lecture to children, usually on dynamics of explosion, with live demonstrations full of bangs and smoke. Crucially, he treated the children as future researchers; I found later that career advice from him, although somewhat non-traditional, was straight to the point:

- while you are young, try everything; avoid limiting choices still open to you;
- but be prepared for a deep, systematic, and specialised study and research;
- however, remember that *the most important thing in our profession is to slip away before cops arrived* (here Lavrentiev was using a popular slang expression): you have to learn to decide when you have to abandon, without undue delay, a line of research which is no longer promising.

> tific knowledge and resort to primitive popularisation of science of the kind that all the standard school courses had suffered from. The other danger was an excessively deep development of a few narrow themes. Having swung between these extreme points, we selected only the most important and essential topics within modern scientific knowledge. As a result, our compulsory courses contain only fundamental knowledge. It turned out that this knowledge was compact, logic of its usage was clear, and the transparency and depth of its inner connections were striking.

Bichenkov based his course on the famous books *The Feynman Lectures on Physics*, which became our principal textbooks in physics. Richard Feynman's rigour of scientific reasoning and militant academic integrity so powerfully professed in his books fitted the ideas of FMSh remarkably well.

In short, physics was taught to us as a concentrated form of *fundamental knowledge* and *criteria* and principles of physical thinking. The same applied to the teaching of mathematics, with the important addition of *mathematical rigour*.

For example, this meant that the course of mathematics covered such fundamentals of analysis as a rigorous theory of continuity and limits, while technical manipulations like solving differential equations or calculations with cross products in vector algebra had been devolved to physics. But I would rather postpone a detailed discussion of syllabus until another occasion.[†]

Academic diversity

Professor Sergei Utyuzhnikov[‡] who read earlier versions of this text, commented that the unique character and excellence of FMSh were achieved only because it was a school not only for young mathematicians but also for young physicists and for chemistry fanatics (and, in my years at the School, there also was an engineering stream, a colourfull soldering iron brigade). Not only mathematics and physics,

[†]I use this opportunity to thank Alexander Shen who provided me with a pack of learning materials from School no. 57, and my French colleague (who prefers to stay anonymous), who gave me links to Lycée Louis le Grand; these two schools make an interesting comparison with the Novosibirsk FMSh.

[‡]`http://www.mace.manchester.ac.uk/aboutus/staff/academic/profile/index.html?staffId=`.

but also chemistry and biology were taught by professional researchers, and at an advanced level. This created an environment of intellectual diversity and stimulated learning by absorbtion, just by talking to each other. Also, the academic diversity of the school moderated competition between children, it opened up many different ways for them to excel.

He also emphasised another point: FMSh was a boarding school; the dormitory and the school building were next to each other connected by a short underground passage. Learning was an integral part of life; the entire day of a student was saturated by mathematics and sciences.

Conclusions

As a summary, I wish to quote an email from Professor Alexander Veselov, a former student and then a teacher and one time Head of Mathematics of The Kolmogorov School:

> I can see that your experience was very similar to mine. I am very sceptical about the relevance of it to the UK reality though. Therefore *I think we should concentrate on two principles, which are most important from my point of view:*
>
> – *A thorough selection process (possibly with summer schools).*
>
> – *The leading role of professional mathematicians (like Kolmogorov and Lavrentiev) with teaching support from PhD students and best Maths undergraduates.*

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Appendix A: Typical questions from selection interviews to Summer School of Novosibirsk FMSh

I picked some of these problems from *Introductory Assignment* of *Gelfand Correspondence Program in Mathematics*[†]. They are part of traditional "mathematical folklore" and many of them have been actually used by me in selection interviews; all of them are set in the genre of "interview problems", they are best told in a face-to-face chat with a child, with the help of some scrawlings on scratch paper.

PROBLEM 1. Which is bigger,

$$\frac{100000001}{100000002} \quad \text{or} \quad \frac{200000001}{200000002}?$$

PROBLEM 2. Some of the inhabitants of the city Boole are liars and always lie, all others always tell the truth. Once ten inhabitants of the city Boole met in a room and each one said: "all the rest of you are liars". How many people in the room were liars?

PROBLEM 3. Each of the equations below is missing a pair of numerators:

(a)
$$\frac{?}{7} - \frac{?}{5} = \frac{1}{35},$$

(b)
$$\frac{?}{5} - \frac{?}{7} = \frac{1}{35}.$$

Assume that the numerators are positive integers. Find as many pairs of numerators as you can. Do not forget that mixed fractions are allowed.

[†] `http://gcpm.rutgers.edu/problems.html`

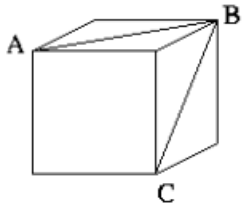
PROBLEM 4. Is the sum

$$1 + 2 + 3 + 4 + \ldots + 98 + 99 + 100$$

even or odd?

Note: The "..." stands for the missing terms, and there are 100 terms in all.

PROBLEM 5. What is the measurement, in degrees, of the angle between the diagonals, AB and BC, of the adjacent faces of the cube?



PROBLEM 6. In a box, there are fresh cucumbers which weigh 100 pounds. Each cucumber is composed of 99% water. After some time, the cucumbers dried out. Now each cucumber is composed of 98% water. How much do the cucumbers weigh now?

PROBLEM 7. The difference of two numbers is 0.01. Is it possible that the difference of their squares is more than 1000?

About the author

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His interests in mathematics, sciences and philosophy have been shaped at the PhMSh, a Preparatory Boarding School of the Novosibirsk University.