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INFORMATION AND COMMUNICATION TECHNOLOGY IN UNIVERSITY LEVEL MATHEMATICS TEACHING

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ABSTRACT

University mathematicians are often selective in their approaches to the use of Information and Communication Technology (ICT) in teaching. Although mathematicians systematically use specialist software in teaching of mathematics, as a means of delivery e-learning technologies have so far been less widely used. We (mathematicians) insist that teaching methods should be subject-specific and content-driven, not delivery-driven. We oppose "generic" approaches to teaching, including the excessively generalist, content-free, one-size-fits-all promotion of IT. This stance is fully expressed, for example, in recent statements from the London Mathematical Society: in the Teaching Position Statement [†] and in the Position Statement of the London Mathematical Society *Use and Misuse of ICT* [‡]. Furthermore, this position is supported by a recent report from the National Union of Students:

> Not every area of study needed or was compatible with e-learning, and so to assume it would grant blanket advantages was not accurate. [§]

This paper is an attempt to explain, at an informal level, this selectivity and its guiding principles. *The paper is addressed to our non-mathematician colleagues* and is not intended to be a survey of the existing software and courseware for mathematics teaching—the corpus of existing solutions is enormous and any technical discussion inevitably involves some hardcore mathematics.

2000 *Mathematics Subject Classification* 97C50 (primary), 97D20 (secondary).

[†] London Mathematical Society. Mathematics degrees, their teaching and assessment.
`http://www.lms.ac.uk/policy/2010/teaching_position_statement.pdf`.

[‡] Use and Misuse of Information and Computer Technology in the Teaching of Mathematics at HE Institutions. Position Statement of the London Mathematical Society. Approved by the LMS Council 25 March 2011.
`http://www.lms.ac.uk/sites/default/files/Mathematics/policy_responses/ICT_statement.pdf`.

[§] Student perspectives on technology—demand, perceptions and training needs. Report to HEFCE by NUS 2010, p. 5.
`http://www.hefce.ac.uk/pubs/rdreports/2010/rd18_10/rd18_10.pdf`.

Disclaimer

Needless to say, all opinions expressed in the present paper are of the author and no-one else. This point needs to be emphasised because this paper contains some material which the author developed for background discussions during the preparation of the Position Statement of the London Mathematical Society *Use and Misuse of ICT* [¶]. The author worked on the present paper in his private capacity, all views expressed here may or may not represent the position of the London Mathematical Society which does not bear any responsibility for the content of this paper.

1. *Selectivity: why?*

University mathematicians are selective in their approaches to the use of ICT in teaching. Our position is not rooted in ignorance or arrogance; on the contrary, I argue that mathematics deserves special treatment not only because of its highly specific cognitive nature, but also because the mathematical community has accumulated much more experience of using computers and ICT in teaching, learning, research and communication than many our colleagues outside of STEM disciplines have attained in their considerably shorter exposure to ICT. We are not special: computer science, physics, many engineering disciplines are in a similar position and should be trusted to use their own tried and tested approaches to ICT. [†]

Historically, mathematicians (and computer scientists) were the first to use ICT in teaching. Even in the era of mainframe computers, green displays and dot matrix printers, some serious work was done in this area (for example, mass generation of random problems of controlled level of difficulty in linear algebra and differential equations) [‡].

¶ Use and Misuse of Information and Computer Technology in the Teaching of Mathematics at HE Institutions. Position Statement of the London Mathematical Society. Approved by the LMS Council 25 March 2011. `http://www.lms.ac.uk/sites/default/files/Mathematics/policy_responses/ICT_statement.pdf`.

† More generally, the thesis that e-learning is only one of many streams of learning and should not be viewed as an universal solution is formalated by many authors, see, for example, T. Franklin, e-learning, b*****-learning and f*****-learning or what is wrong with e-learning, `http://www.franklin-consulting.co.uk/LinkedDocuments/e-learningandb-learning.doc`.

‡ A useful and representative guide to professional mathematicians' assessments of, and comments on, the developments in computer based learning of mathematics from 1988 to 1994 can be found in K. Devlin and N. Wilson, Six-Year Index of "Computers and Mathematics", *Notices AMS* 42 no. 2 () 248–254. `http://www.ams.org/notices/199502/devlin.pdf`.

University mathematicians (and not only researchers, but all mathematicians who teach all kinds of mathematics in universities) form a professional community; it is global and transcends national boundaries, but at the same time it is closely knit and connected in an efficient network. And yes, mathematicians were some of the first to use email, too; it started at least 30 years ago; the Internet at the time existed as a set of ftp sites and was unknown outside of mathematics, physics and computer science departments. Therefore a discussion of accumulated experience, tradition and collective wisdom of the mathematics community is well justified.

In short, the mathematics community has experience and knowledge of what can and cannot be done with computers. In that respect, we are not alone: to name a few, similar experiences have been accumulated in computer science, or, say, in language teaching. But we differ from our colleagues in some other subject disciplines who are still on a path of discovery. And I sincerely hope that ICT solutions that do not work in mathematics teaching can be happily used elsewhere. However whilst many available products are suitable for many disciplines, they are unsuitable and unworkable for mathematics.

2. *A case study: TeX*

In the late 1970s, the great mathematician and computer scientist Donald Knuth launched a revolution in scientific communication by creating TeX [†, ‡], a cross-platform computer language for typesetting mathematical texts. In one step, he brought mundane mathematical scribbles—not only research papers but also lecture notes, exercise sheets, seminar handouts—to the highest reaches of typographic art. Since the early 1990s, TeX and its dialect, LaTeX [§, ¶], have been international standards for mathematical typesetting. But the routine everyday use of TeX and LaTeX in teaching in every mathematics department remains unnoticed and unappreciated by the wider education community. This is unfortunate, because TeX/ LaTeX is a pedagogical success story: it allows us to present even the most complicated mathematical formulae as structured and logically justified shapes, optimised for visual processing by the human eye and brain. After all,

[†] D. E. Knuth, TeX and Metafont: New directions in typesetting. The American Mathematical Society and Digital Press. Stanford, 1979

[‡] D. E. Knuth, The TeXbook. Addison-Wesley, Reading, 1986.

[§] L. Lamport, LaTeX—A document preparation system—User's guide and reference manual. Addison-Wesley, Reading, 1985.

[¶] `http://www.latex-project.org/`.

> Typography may be defined as the craft of rightly disposing printing material in accordance with specific purpose; of so controlling the type as to aid to the maximum the reader's comprehension of the text. [||]

TEX succeeded in part because Donald Knuth spent years studying the millennia long tradition of calligraphy and the art of typesetting (five centuries old) [†,‡].

When speaking about commercially available software systems for teaching and learning in higher education, we can safely conclude that in 95% of these products the mathematical presentation lags 20 years behind TEX ; their developers have not done their homework with the same care as Donald Knuth did his. Too frequently, ICT developers and promoters of e-learning invite mathematicians back to the Stone Age.

The whole of this paper is written in LATEX; for those who have never seen how LATEX typesets mathematical formulae, Figure 1 shows an example of LATEX output in font type, size, and column width suitable for viewing on narrow screens of iPhones (some of my students indeed use their iPhones for access to bite-sized learning materials like exercise sheets—although iPhones are less convenient for reading more substantial pieces of text like lecture notes).

Figure 2 demonstrates spatial positioning of complicated formulae.

For dyslexic students, one can easily meet disability consultants' recommendations by setting up lecture notes in landscape mode, double line spacing and a huge sans serif font, see Figure 3.

If further enhancement is needed, a font handling facility of LATEX allows the incorporation of specialist fonts like Lexia Readable [§] for dyslexic readers, but so far my students, when given a choice, have preferred the standard Computer Modern font in its sans serif version.

3. *Students matter*

I first started teaching computer-based courses in 1995—and was not a pioneer since I used off-the-shelf software packages and associated textbooks,

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[||] S. Morison, First Principles of Typography. Cambridge University Press, 1951. Quoted from: R. Lawrence, Maths = Typography? TUGboat 24 no. 2 (2003).

[†] D. E. Knuth, Mathematical typography, Bull. Amer. Math. Soc. 1 no. 2 (1979), 337–372

[‡] D. Knuth, Digital Typography. Cambridge University Press, 1999 (reissue edition). ISBN-10: 1575860104; ISBN-13: 978-1575860107.

[§] `http://www.k-type.com/?p=884`.

Here is Schrödinger's Equation:

$$i\hbar\frac{\partial}{\partial t}\Psi = \hat{H}\Psi,$$

and here is Stokes' Theorem:

$$\int_{\Omega} d\omega = \oint_{\partial\Omega} \omega,$$

and this is an infinite product expansion for the Gamma function:

$$\begin{aligned}\Gamma(z) &= \lim_{n\to\infty} \frac{n!\, n^z}{z\,(z+1)\cdots(z+n)} \\ &= \frac{1}{z}\prod_{n=1}^{\infty}\frac{\left(1+\frac{1}{n}\right)^z}{1+\frac{z}{n}}.\end{aligned}$$

FIGURE 1. *Example of LaTeX output optimised for viewing on narrow screens of hand handled mobile devices.*

$$\begin{aligned}\frac{1+\sqrt{5}}{2} &= \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}} \\ &= 1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cdots}}}}\end{aligned}$$

FIGURE 2. *This is how LaTeX handles nested roots and continued fractions.*

$$\mathsf{\Gamma(z) = \frac{1}{z}\prod_{n=1}^{\infty}\frac{\left(1+\frac{1}{n}\right)^z}{1+\frac{z}{n}}.}$$

FIGURE 3. *Some dyslexic students prefer formulae typeset in a large sans serif font .*

already developed by my colleagues elsewhere, and then tested, published, and reviewed. In my courses, the media of electronic communication were web pages and email. In one course (on mathematical logic), student assignments were marked automatically, by a computer (I used an early version of TARSKI'S WORLD [¶]; an improved version is now available as [‖]). In another course, on number theory and cryptography [**], students (whose identities were hidden online under aliases) were invited to attack each others' ciphers, and the ensuing fights provided the most rigorous form of assessment. I had a chance to observe pedagogical and psychological *mise en scènes* evolving from the constraints of a technological set-up.

My experience makes me to believe that mathematics students differ from the general student population: in mathematics, students' attitudes to ICT are much more diverse and complex. It sounds paradoxical, but quite a few mathematics students dislike computers (for otherwise they would study computing and computer science).

At the other end of the spectrum, we have the beginner hackers (not very experienced ones, so-called "script kiddies" [†]); for whom the ICT solutions offered at the university are primitive and boring. (In mass media, the term "hacker" has negative connotations; among computer enthusiasts, "hacker" is a term of respect, it means someone who can do clever tricks ("hacks") with computers and software [‡,§].) In my number theory and cryptography course, hackers had a plenty of opportunities to show themselves and their skills. Also we have among our students a number of adrenalin charged gamers who cannot wait, for purely physiological reasons, if a VLE hangs for a few seconds. Comparing mathematics with other disciplines, I make an educated guess that gamers can be found, say, in Humanities—but not that many script kiddies.

[¶] J. Barwise and J. Etchemendy, The Language of First-Order Logic: Including the IBM-compatible Windows version of Tarski's World 4.0. Cambridge University Press, 1993. ISBN-10: 0937073903; ISBN-13: 978-0937073902.

[‖] D. Barker-Plummer, J. Barwise and J. Etchemendy, Tarski's World. Chicago University Press, 2008. ISBN-10: 1575864843; ISBN-13: 978-1575864846.
`http://ggww.stanford.edu/NGUS/tarskisworld/`.

[**] This my course was close in spirit to the book of P. J. Giblin, Primes and Programming, Cambridge University Press, 1993. ISBN-10: 0521409888; ISBN-13: 978-0521409889. Instead of bespoke code in PASCAL, I was using off-the-shelf routines of MATLAB. Nowadays, I would use the book by W. Stein, Elementary Number Theory: Primes, Congruences, and Secrets, Springer, 2008, and the open source software package SAGE.

[†] See an explanation of the term "script kiddies" in
`http://www.catb.org/~esr/jargon/html/S/script-kiddies.html`.

[‡] D. Thomas. Hacker Culture. University of Mineapolis Press, 2002.

[§] C. Legg, Hacking: The performance of technology? Techné 9 no. 2 (2005), 151–154. Available at
`http://waikato.academia.edu/CathyLegg/Papers/209879/Hacking--The-Performance-of-Technology-`.

In the cryptanalytic battles which I mentioned above, students revealed their psychological positions in their choice of aliases. Over the years, I had in my class several girls who called themselves Piglet. Alas, the outcome of an encounter between Piglet and, say, Darth Vader (a gamer) was entirely predictable. Interestingly, Tigger (another girl and a friend of Piglet) ferociously and successfully fought back. However, Tigger had fallen under a sustained onslaught from Xterminator (a script kiddie) who, unsatisfied by tools provided in the course, downloaded from the Web and compiled an industrial strength C++ code. Of course, my primary duty as a teacher was to give Piglet and Tigger not only technical help, but also moral support and encouragement.

These experiences made me sensitive and attentive to personal attitudes of my students to computers and ICT, and led me to believe that there is no one universal solution that suits all students.

Studies of students' attitudes to ICT already exist, and [†] provides a useful survey. A recent report from the National Union of Students expresses a summarised students' opinion in a very direct an unambiguous way. I quote two points which match my personal observations.

One point is that students want to have choice and want to be in control:

> Students prefer a choice in how they learn—ICT is seen as one of many possibilities, alongside part-time and traditional full-time learning, and face-to-face teaching. [‡]

> Students could see some advantages to an e-learning approach. If it were presented as an option, as opposed to an obligation, it would avoid onerous undertones. [§]

Another point is that—surprise surprise—content matters for students more than delivery:

> Participants expressed concerns over "surface learning" whereby a student only learns the bare minimum to meet module requirements—this behaviour was thought to be encouraged by ICT: students can easily skim-read mate-

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† Learner acceptance of on-line learning and e-learning,
`http://wiki.alt.ac.uk/index.php/Learner_acceptance_of_on-line_learning_and_e-learning`.

‡ Student perspectives on technology—demand, perceptions and training needs. Report to HEFCE by NUS 2010, p. 3.
`http://www.hefce.ac.uk/pubs/rdreports/2010/rd18_10/rd18_10.pdf`.

§ *ibid.*, p. 5.

> rial online, focusing on key terms rather than a broader base of understanding. [¶]

Students matter, and their opinion should matter, too.

4. *What we want: windows in mathematical worlds*

Let me formulate in one word an important shared key feature of ICT that finds modern uses in direct teaching of mathematics: this word is *virtualisation*. A computer is useful if it creates a new (virtual) reality that cannot be created by other means. [†] In mathematics, the word "reality" includes the ideal Platonic world of mathematical objects and structures. MATLAB [‡], MAPLE, MATHEMATICA—mathematics software packages widely used in undergraduate teaching—are windows into this Platonic world. As a rule, software that provides such windows needs a powerful mathematical engine. MATLAB, MAPLE, MATHEMATICA and statistics packages such as SPSS (commercial) and R (open source and free) are not just toys for learning—they are professional research tools; mastering them is a valuable transferable skill for graduates seeking employment in mathematics-intensive industries. This list is now incomplete without mentioning a newcomer, an open source package SAGE—I talk more about it later in this paper.

I can give less known and more specialised examples, like the already mentioned TARSKI'S WORLD—an expertly crafted courseware package for learning mathematical logic, and the wonderful visualisation and experimentation tools for elementary geometry, CINDERELLA [§] and GEOGEBRA [¶].

Assessment of mathematics learning software inevitably involves a mathematical characterisation of its built-in mathematical world. For example, it matters that the interface language of TARSKI'S WORLD is "interpreted"

[¶] *ibid.*, p. 5.

[†] A metaphor of mathematics as a virtual reality game is perhaps best formulated by Anna Sfard (A. Sfard, Symbolizing mathematical reality into being—or How mathematical discourse and mathematical objects create each other. In Symbolizing and Communicating: Perspectives on Mathematical Discourse, Tools and Instructional design (P. Cobb et al., eds.). Mahwah, NJ: Erlbaum, 1998, pp. 37–98.).

[‡] A list of available learning resources—far from being complete—can be found at `http://www.mathworks.com/matlabcentral/linkexchange/?term=tag:"mathematics"`.

[§] An earlier version of CINDERELLA can be downloaded for free from `http://cinderella.de/tiki-index.php?page=Download+Cinderella+1.4&bl`.

[¶] GEOGEBRA is free and open source, `http://www.geogebra.org/cms/`.

(in some specialised meaning of this word as used in logic)—and serious implications of this fact for teaching logic with Tarski's World had been pointed out in one of the first reviews of the package [||]—written in 1989!

If you wish to get hands-on experience, I invite you to have a look at Cinderella and GeoGebra—they are available for free, and their uncluttered minimalistic interfaces provide for an immediate usability. The next two paragraphs use Cinderella and GeoGebra to give a sample of a mathematician's approach to probing and testing the software; they can be skipped in the first reading.[†]

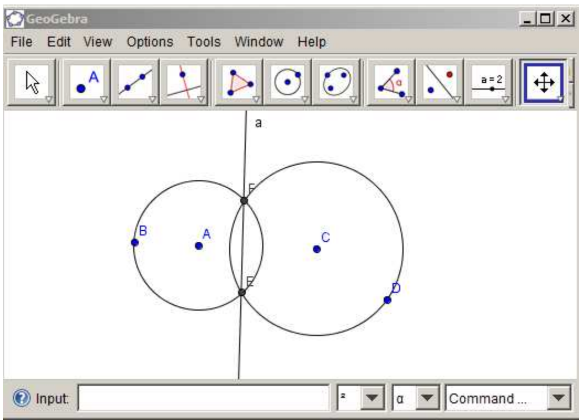

Figure 4. GeoGebra*: a line through the intersection points of two circles.*

It is interesting to compare the behaviour, in Cinderella and GeoGebra, of a simple interactive diagram: two intersecting circles of varying radii and the straight line determined by their points of intersection. In GeoGebra, when you vary the radii or move the centers of the circles and make the circles non-intersecting, the line through the points of intersection disappears—exactly as one should expect; Figures 4 and 5.

In Cinderella, the line does not disappear, it moves following the movements of the circles, always separating them; when circles touch each other and start to intersect again, the line turns to be, again, the common tangent line of two circles or, in the case of two intersecting circles, the line through the points of intersection. This line is called the *radical axis* of the two circles; Figures 6 and 7.

To a mathematician, the behaviour of this diagram suggests that the underlying mathematical structure of GeoGebra is the real Euclidean plane.

[||] W. Hodges, *Review of J. Barwise and J. Etchemendy,* Tarski's World *and* Turing's World, Computerised Logic Teaching Bulletin 2 (1) (1989) 36–50.

[†] For the sake of formal completeness I have to mention other elementary geometry pachages: Cabri `http://www.cabri.com/` and The Geometer's Sketchpad `http://www.dynamicgeometry.com/`.

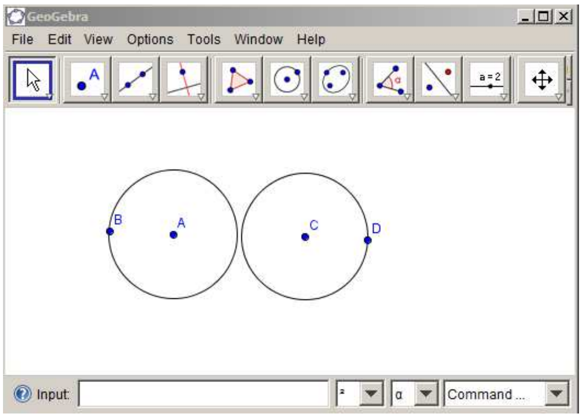

FIGURE 5. GEOGEBRA*: the line through the intersection points of two circles disappeared after the circles are disengaged.*

> In CINDERELLA, the underlying structure is the complex projective plane; what we see on the screen is just a tiny fragment of it, a real affine part. The radical axis of two non-intersecting circles is the real part of the complex line through two complex points of intersection. The intersection points of two real circles are complex conjugate, the line is invariant under complex conjugation and therefore is real and shows up on the real Euclidean plane. For a mathematician, this is a strong hint that CINDERELLA could work better than GEOGEBRA in accommodating non-Euclidean geometries: elliptic and hyperbolic (the Lobachevsky plane) since they happily live in the complex projective plane. But perhaps there may be pedagogic situations where the real Euclidean plane might be a safer tool for some students than the complex projective plane, even if the difference is undetectable at first glance.

Java applets providing interactive geometric diagrams are relatively simple at a technical level but could provide wonderful enhancment; a beautiful example is an on-line interactive version of Euclid's *Elements* [†].

5. *Understanding of (virtual) worlds*

Mathematics is a virtual world in itself (actually, a whole universe of virtual worlds), and since the universe of mathematics is ultimately an image of the physical universe, it has an immense complexity. Even if a student

[†]D. E. Joyce, Euclid's *Elements*,
`http://aleph0.clarku.edu/~djoyce/java/elements/elements.html`.
Accessed 30 May 2011.

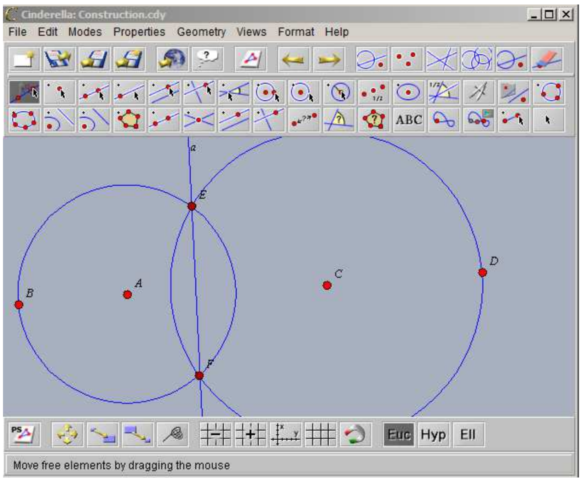

FIGURE 6. CINDERELLA*: a line through the intersection points of two circles.*

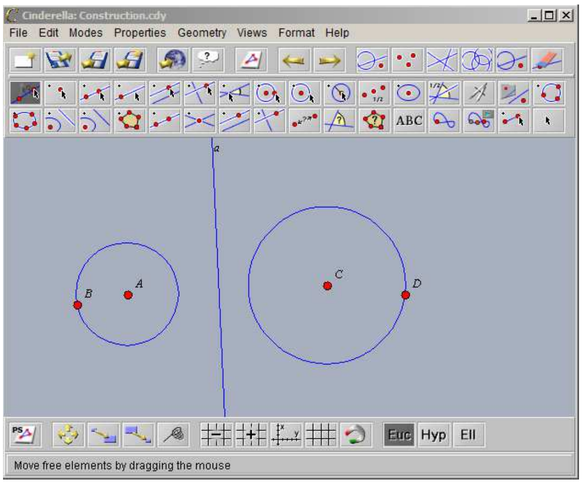

FIGURE 7. CINDERELLA*: the line through the intersection points of two circles (their* radical axis*) does not disappear after the circles are disengaged.*

has a comfortable view at this universe from a window provided, say, by MATHEMATICA, it does not meant that he or she can easily understand it.

What follows is a fragment from a recent email from my (first year) student:

```
>
> for Q A5 of the 2009 paper qb
>
> isnt the igen vector -1,1 ?
>
```

```
> this is what i get from,
> using an online calculator found here
> www.arndt-bruenner.de/mathe/scripts/engl_eigenwert2.htm
>
```

Yes, indeed, if the matrix

$$\begin{bmatrix} 1 & 2 \\ -2 & -3 \end{bmatrix}$$

from the exam is entered into the online calculator [†], it produces an eigenvector $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$; my answer that was questioned by my student was $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Alas, computation, especially quick user-friendly automated computation, cannot replace understanding.

And let us have a look at the upper end of the market, WOLFRAM MATHEMATICA ONLINE INTEGRATOR [‡]. It integrates *everything*. But enter an innocuously looking function

$$\frac{1}{\sqrt{1+x^3}}:$$

the INTEGRATOR will return an answer that 90% of graduates from mathematics departments in this country will not be able to interpret [§]:

$$\begin{aligned}
&\int \frac{dx}{\sqrt{1+x^3}} \\
&= \frac{2\sqrt[6]{-1}}{\sqrt[4]{3}\sqrt{1+x^3}} \\
&\qquad \cdot \sqrt{-(-1)^{1/6}\cdot((-1)^{2/3}+x)} \\
&\qquad \cdot \sqrt{1+\sqrt[3]{-1}x+(-1)^{2/3}\cdot x^2} \\
&\qquad \cdot F\left(\sin^{-1}\left(\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right), \sqrt[3]{-1}\right),
\end{aligned}$$

where $F(\phi,k)$ is the elliptic integral of the first kind.

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† Calculate eigenvalues and eigenvectors, `http://www.arndt-bruenner.de/mathe/scripts/engl_eigenwert2.htm`.

‡ `http://integrals.wolfram.com/`

§ Actually, an experienced colleague wrote to me: "*I reckon you're too optimistic. I would have said 90% would fail to explain the* $(-1)^{5/6}$*, and of the 10% left not more than 1 in 10 would be able to tackle the Elliptic function*".

6. *The unity of research and teaching*

One interesting feature of MATLAB, MAPLE, MATHEMATICA and SPSS is that they were originally designed and developed for research purposes and only later fed into university teaching—mostly by mathematicians who transferred to their teaching the skills developed in their research. It was the mathematics research community who acted as a driver of technological change in mathematics teaching. This example, even taken on its own, demonstrates the futility of erecting a fence between mathematics research and mathematics teaching.

The situation with specialised teaching-only software packages is even more instructive. Returning to one of my case studies, TARSKI'S WORLD, I wish to comment that one of its authors—and the initiator of the project—was Kenneth Jon Barwise, a prominent mathematician, philosopher and logician.

Development of TARSKI'S WORLD and other programs that became part of the courseware package *Language, Proof and Logic* [†]: FITCH, BOOLE, GRADE GRINDER, required not only a pioneering re-assessment of methodology of teaching mathematical logic [‡], but also the creation of a new direction in mathematical logic itself, [§], *heterogeneous reasoning*, which formed the core of the computer algorithms implemented as courseware. The first reviewers of TARSKI'S WORLD [¶, ‖] were fully aware of mathematical difficulties that its authors had to overcome.

The work of Jon Barwise and his collaborators is a manifestation of a phenomenon specific to mathematics: the central role of *didactic transformation*, that is, *mathematical reworking* of teaching material into a form suitable for students' consumption. The term *transformation didactique* was coined in 1852 by French philosopher Auguste Comte [**] and is well

[†] J. Barwise and J. Etchemendy, Language, Proof and Logic. CSLI Publications, 2003. Distributed by the University of Chicago Press. ISBN 157586374X.

[‡] J. Barwise and J. Etchemendy, Computers, visualization, and the nature of reasoning, in The Digital Phoenix: How Computers are Changing Philosophy (T. W. Bynum and J. H. Moor, eds.). Blackwell, 1998, pp. 93–116.

[§] S.-J. Shin, Heterogeneous reasoning and its logic, The Bulletin of Symbolic Logic 10 no. 1 (2004) 86–106.

[¶] G. Boolos, Review of Jon Barwise and John Etchemendy, Turing's World and Tarski's World, J. Symbolic Logic 55 (1990) 370–371.

[‖] D. Goldson and S. Reeds, Using programs to teach Logic to computer scientists, Notices Amer. Math. Soc. 40 no. 2 (1993) 143–148.

[**] A. Comte, Catéchisme positivist. 1852.

known in French education studies [††] but remains unused in English-language literature on education. Hyman Bass, a prominent mathematician and a champion of mathematics education, picked up from his French colleague Jean-Pierre Kahane [‡‡] the term "didactic transformation" and an explanation of its role in relations between mathematics and mathematics education:

- In no other living science is the part of presentation, of the transformation of disciplinary knowledge to knowledge as it is to be taught (*transformation didactique*) so important at a research level.
- In no other discipline, however, is the distance between the taught and the new so large.
- In no other science has teaching and learning such social importance.
- In no other science is there such an old tradition of scientists' commitment to educational questions.

You can read more about didactic transformation in my paper [†] or in Chapter 9 of my book *Shadows of the Truth* [‡]. Here I will add only that, in Barwise's case, didactic transformation took the form of serious cutting-edge mathematical research in logic which was than fed into state-of-the-art software development.

Even the roots of dynamic geometry packages aimed at teaching of elementary Euclidean geometry can be traced back to the hard core research. This is a testimony from Judah Schwartz, creator (in early 1980's) one of the first interactive geometry packages, THE GEOMETRIC SUPPOSER:

> In the mid-1960s, while on the research staff of the Lawrence Radiation Laboratory in California, two colleagues and I developed a series of computer-generated motion picture films that depicted graphically the collisions of subatomic particles as described by formal mathematical machinery of quantum mechanics. I became interested in the potential of the computer to make accessible representations of spatial and temporal whose natural distance and time scale lay well outside the ken of human sensory apparatus. My colleagues and I pursued these efforts for a while but soon concluded that the true utility of computers to help people with mathematical abstraction

[††] Y. Chevallard, La transposition didactique—Du savoir savant au savoir enseigné. La Pensée sauvage, Grenoble, 1985.

[‡‡] H. Bass, Mathematics, mathematicians, and mathematics education, Bull. Amer. Math. Soc. 42 no. 4 (2005), 417–430.

[†] A. V. Borovik, Didactic transformation in mathematics teaching, in The Teaching-Research Interface: Implications for Practice in HE and FE (Muir Houston, ed.). Higher Education Academy Education Subject Centre, Bristol, 2008, pp. 30–35. ISBN 978-1-905788-81-1.

[‡] A. V. Borovik, Shadows of the Truth: Metamathematics of Elementary Mathematics.

would lie in the interactivity and control that the soon-to-come microcomputer would offer. [§]

7. *A parallel universe: computer assisted language learning*

I have already mentioned language teaching as an interesting parallel to mathematics; indeed, CALL, Computer Assisted Language Learning, as an remarkable development outside mathematics which in some aspects mirrors CAL in mathematics. Software packages for learning languages frequently involve powerful engines that support a "virtual interlocutor", a software device that listens to the learner, recognises and analyses the learner's speech, corrects errors and gives feedback. Creation of such tools would be impossible without decades of development of computational and mathematical linguistic. [†]

CALL benefited from interest and attention of computer scientists to linguistics which started in 1950-s and 1960-s, when machine translation between human languages was a Holy Grail of rapidly developing computer science. Grammar correcting software for written exercises in foreign languages which started to appear in 1980-s was an out-spun of the earlier attempts to develop natural grammar parsers for machine translation. Speech recognition modules were originally developed for wider non-academic applications, which, in their turn, had generous funding from the military and the industry. A frequent complaint about language learning software is that it is expensive; this is not surprising, given the huge cost of development.

8. *Paradoxical economics of education*

So, mathematicians have developed, and systematically use, specialist software in direct teaching of mathematics—and find it very useful.

However, as means of delivery of mathematics teaching, ICT and e-learning technologies have so far been unable to meet our expectations. There are several reasons for this.

[§] J. L. Schwartz, M. Yerushalmy, and B. Wilson. 1993. The Geometric Supposer: what is it a case of? Routledge, 1993. p. 4.

[†] Some fragments of history of these developments are told in in B.-H. Juang and L. R. Rabiner, Speech Recognition, Automatic: History, in Encyclopedia of Language and Linguistics (K.Brown, ed.). 2nd edition, Elsevier, 2006, pp. 806–819. See also M. Kay, A life of language, Computational Linguistics 31 no. 4 (2006), 425–438.

One reason is that our expectations are high. Due to the level of sophistication already achieved, say, in MATLAB / MATHEMATICA / MAPLE or in TeX / LaTeX, mathematicians' demands for functionality of ICT are high and are not met by many software packages and VLEs currently promoted in British universities.

At a neurophysiological level, teaching / learning mathematics is a communication between two brains. It is best done one-to-one, or in a small group. Large class lectures are an unhappy compromise with economic necessity. From a pedagogical point of view, the right alternative to a large class lecture is not streaming-on-demand of video recordings; the true alternative is a small class lecture. Unfortunately, this alternative in most cases is financially infeasible. Collaborative on-line small groups provide some interesting possibilities, but students themselves insist that ICT should be a supplement, not a replacement of the face-to-face teaching:

> Motion 306, passed at the April 2010 NUS National Conference states that:
> [...]
> 4. The provision of e-learning should be utilised as a tool for learning, in all institutions, but that should not merely be used as a method of reducing costs and should be in conjunction with, not instead of, other face-to-face teaching methods.
> 5. Technology should complement good teaching, allowing students to benefit from the additional value of e-learning but should not be used as a substitute for face-to-face contact and good teaching. [†]

There is a need to assess the efficiency of particular methods of teaching not only from the pedagogical, but also from a socio-economic point of view. Of course, "generic" technologies are very tempting to policy-makers because of their promise (mostly unrealistic) of economies of scale. Large class teaching is frequently mentioned as obvious point of application of ICT. But I would like to point that large class teaching is already, by default, under-resourced teaching. It is futile to expect further savings brought by use of expensive technology.

At a more fundamental level, it would be wrong to reduce the all-important discussion of learning and teaching to deciding the choice of the cheapest variety of margarine as a substitute for butter.

And, last but not least, at the socio-economic level relations between mathematics and information technology are also paradoxical.

Mathematics, by its nature, is an open source phenomenon. A powerful

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[†] Student perspectives on technology—demand, perceptions and training needs. Report to HEFCE by NUS 2010, p. 18.
`http://www.hefce.ac.uk/pubs/rdreports/2010/rd18_10/rd18_10.pdf`.

formulations of this principle belongs to Joachim Neubüser, the initiator and leader of the GAP project, perhaps one of the most successful community projects in experimental mathematics [‡].

> You can read Sylow's Theorem and its proof in Huppert's book in the library without even buying the book and then you can use Sylow's Theorem for the rest of your life free of charge, but—and for understandable reasons [. . .]—for many computer algebra systems license fees have to be paid regularly for the total time of their use. In order to protect what you pay for, you do not get the source, but only an executable, i. e. a black box. You can press buttons and you get answers in the same way as you get the bright pictures from your television set but you cannot control how they were made in either case.
>
> With this situation two of the most basic rules of conduct in mathematics are violated: In mathematics information is passed on free of charge and everything is laid open for checking. Not applying these rules to computer algebra systems [. . .] means moving in a most undesirable direction. Most important: Can we expect somebody to believe a result of a program that he is not allowed to see? [†]

It is almost a rule that open source software systems are friendlier to mathematics; perhaps this could be explained by the social and cultural background of the open source movement. Indeed, Neubüser's words

> Can we expect somebody to believe a result of a program that he is not allowed to see?

is a war cry and a cultural paradigm shared by mathematicians (who apply it, in the first instance, to proofs of their theorems) and by the open source warriors.

A good illustration of this principle can be found in a comparison between MOODLE, a free open source VLE (it provides for a decent rendering of LaTeX) and proprietary VLEs, some of which are completely unfit for use in mathematics courses.

Developers of quality proprietary software for mathematics and statistics (like MATLAB / MATHEMATICA / MAPLE, SPSS) have taken reasonable care to allow the users a sufficient degree of freedom in tinkering with the interface (and, at least in the case of MATLAB—with the computational core, too—MATLAB smoothly incorporates bespoke Fortran and C code).

[‡] GAP – Groups, Algorithms, Programming – a System for Computational Discrete Algebra, `http://www.gap-system.org/`.

[†] J. Neubüser, An invitation to computational group theory. Invited talk at the conference 'Groups St Andrews' at Galway 1993. Available in DVI:
`http://www.gap-system.org/Doc/Talks/cgt.dvi`
and PostScript:
`http://www.gap-system.org/Doc/Talks/cgt.ps`.

Also, MATLAB / MATHEMATICA / MAPLE allow the export of results (both symbolic and graphic) in formats directly usable in TEX / LATEX documents.

And R, a very popular statistics package, is a GNU licensed open source product.

TEX, the true and unsurpassed masterpiece of the art of computer programming, is faced with a strange fate: it somehow does not show up on the radar of promoters of ICT for higher education. I believe this has a very simple explanation: TEX is free—thanks to the generosity of Donald Knuth—and open source. It exists like the air that we breath. For that reason TEX remains unadvertised and is not promoted, and therefore goes unnoticed by university administrators who make decisions about the acquisition of ICT products. Paradoxically, these are the same administrators who hold the purse strings and are apparently on the quest for the cheapest ICT solutions. I conjecture that MOODLE is also disadvantaged by being free, not promoted by vested commercial interests, and therefore may be less visible in the market. Judging by statistics of the use of VLEs assembled in a recent report [†], MOODLE remains more popular than its commercial rivals, but is loosing grounds as the *main* (that is, approved by university administration) VLE used in universities.

We have to make free open source options visible—this will allow them to compete with commercial for-profit products. The problem is wider and concerns not only software and ICT, but also textbooks. Perhaps an open-source model supports "niche" disciplines better than a commercial model; indeed, in words of Gary Hall,

> Many publishers have decided to focus on introductions and readers for the relatively large (and so more profitable for their shareholders) first year undergraduate "core course" markets, and hardly produce books for second-and-third-year students, let alone research monographs or even edited collections of original scholarship aimed at postgraduates and other researchers, at all. [‡]

I was teaching one of my lecture courses using an open source GNU

[†] Tables 3.4b and 3.4c in T. Browne, R. Hewitt, M. Jenkins, J. Voce, R. Walker, and H. Yip. 2010 Survey of Technology Enhanced Learning for higher education in the UK. Universities and Colleges Information Systems Association, 2010.
http://www.ucisa.ac.uk/groups/ssg/~/media/groups/ssg/surveys/TELsurvey2010_FINAL.ashx,
retrieved 14 August 2011. See also a blog post by M. Feldstein, The Evolving LMS Market, Part I.
http://mfeldstein.com/the-evolving-lms-market-part-i/,
posted 21 December 2010, retrieved 12 August 2011. Apparently LMS means "Learning Management System".

[‡] G. Hall, Digitize This Book! The Politics of New Media, or Why We Need Open Access Now. Minneapolis: University of Minnesota Press, 2008; p. 42.

licensed textbook [§]. Besides pedagogical reasons, my decision was motivated by new functionality provided by open source textbooks: availability of source LaTeX files gives, for example, a possibility of global changes in the text, say, a uniform change of notation over the entire textbook. In a properly written LaTeX source file, it is achieved by changing one line in the preamble. The need for change in notation rarely arises in most disciplines, but is quite common in mathematics. Availability of the source file resolves a number of issues of disability support—a change of one line in the preambule of the source file allows a lecturer to change size, type, colour of fonts used, switch to the landscape mode, etc.

It is a social imperative of our challenging times: open source teaching suits publicly funded universities best. But, because it is not promoted by commercial interests, it needs its champions. The textbook which I was using had found an unexpected champion in Arnold Schwarzenegger. The book is endorsed by the Free Digital Textbook Initiative run by California Learning Resources Network [†]. CLRN was funded by the state of California. California was experiencing financial difficulties, and the webpage of the Free Digital Textbook Initiative proudly displayed a message from then Governor of California Arnold Schwarzenegger:

> This initiative will ensure our schools know which digital textbooks stand up to California's academic content standards—so these cost-effective resources can be used in our schools to help ensure each and every student has access to a world-class education.

The state of Texas launched a similar initiative [‡]; together, the states of California and Texas control the market of high school textbooks in the USA. At this mighty background, Washington State's initiative to provide open textbooks for the eighty highest-enrollment courses in their community college system [§] looks modest but is interesting because of its very concrete targets. Also, public depositories of open source textbooks like

[§] J. Hefferon, *Linear Algebra*, available for free download from `ftp://joshua.smcvt.edu/pub/hefferon/book/book.pdf`.

[†] `http://www.clrn.org/fdti/`.

[‡] A. Vance, $ 200 Textbook vs. Free. You Do the Math. New York Times, 31 July 2010. `http://www.nytimes.com/2010/08/01/technology/01ping.html?_r=1&emc=eta1`.

[§] Washington State Board for Community and Technical Colleges, Washington State Student Completion Initiative, `http://www.sbctc.ctc.edu/college/e_studentcompletioninitiative`.

CURRIKI [¶] or a South African website *Free High School Science Texts* [‖] are becoming more prominent and influential.

Some of the existing repositories of open education resources [†] and programmes for their development [‡] are still not sufficiently representative and not sufficiently selective. A systematic peer review is badly needed; perhaps it can be provided by greater involvement of subject specialists and their learned societies.

9. *Integration:* SAGE

No discussion of ICT in mathematics teaching would be complete without mentioning SAGE [§]; it is described by its developers as

> a free open-source mathematics software system licensed under the GPL. It combines the power of many existing open-source packages into a common PYTHON-based interface.

SAGE smoothly integrates a number of well-known packages, including GAP (already mentioned in this paper), R, SINGULAR—among almost 100 others [¶].

But SAGE is not only about integrating various available open source mathematics packages. It aims to provide a platform for computational mathematics, enabling users to develop new packages based on the library. In several areas, such as number theory and algebraic combinatorics, there is considerable functionality provided only in SAGE. Hopefully there will be more in the future.

I refer the reader to the explanation of philosophy and design decisions behind Sage in Burçin Eröcal's and Willian Stein's paper prepared for the The Third International Congress on Mathematical Software [‖], to docu-

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¶ `http://www.curriki.org/xwiki/bin/view/Main/WebHome`.

‖ `http://www.fhsst.org/`.

† Such as `http://www.oercommons.org/`.

‡ Such as as the one run by HEFCE and JISC, `http://www.jisc.ac.uk/oer`.

§ W. A. Stein et al. Sage Mathematics Software (Version 4.5.2), The Sage Development Team, 2009, `http://sagemath.org/`.

¶ B. Eröcal, Sage – connecting mathematical software, 2010. `http://erocal.org/burcin/talks/sage-kl_talk.pdf`.

‖ B. Eröcal and W. A. Stein, The Sage Project: Unifying Free Mathematical Software to Create a Viable Alternative to Magma, Maple, Mathematica and MATLAB, `http://wstein.org/papers/icms/icms_2010.pdf`.

mentation [**], and to list of textbooks (mostly free and open source) based on SAGE [††]. You may try SAGE online [‡‡].

A comprehensive range of functions and open source nature of SAGE allows formulation of ambitious projects like OUTMOST (Undergraduate Teaching in Mathematics with Open Software and Textbooks) aimed at conversion of

> existing open textbooks into web-based electronic texts that integrate traditional mathematical exposition with SAGE code and hands-on demonstrations. [†]

This aim is proposed to be achieved

> by integrating SAGE into existing open textbooks and other curricular materials, placing the full computational power of Sage *directly into a student's text*, usable at all times and from anywhere simply via a web browser. [‡]

Such developments as SAGE and OUTMOST deserve to be closely followed.

10. *Computer Aided Assessment*

Online computer-aided assessment (CAA) is a big issue for those of us who want to formatively assess 350+ size classes without using very bland questions. In particular, assessment systems should allow easy and unconstrained entry of mathematical formulae and be able to interpret their meaning. Some obstacles to that are discussed by Chris Sangwin [§] and (mathematical!) solutions offered in [¶].

Another current difficulty is that CAA is dominated by multiple choice questions. As Sangwin points out, multiple choice questions go against the grain of mathematics: most procedures in mathematics are non-symmetric,

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** `http://sagemath.org/doc/`.

†† `http://sagemath.org/`

‡‡ `http://www.sagenb.org/`.

† OUTMOST Project Summary,
`http://buzzard.ups.edu/private/nsf-ccli-summary.pdf`.

‡ OUTMOST Project Description,
`http://buzzard.ups.edu/private/nsf-ccli-proposal.pdf`.

§ C. Sangwin, Assessing elementary algebra with STACK, 2006.
`http://www.open.ac.uk/cetl-workspace/cetlcontent/documents/4607d31d634fd.pdf`.

¶ C. J. Sangwin and P. Ramsden. Linear syntax for communicating elementary mathematics. J. Symbolic Computation, 42 no. 9 (2007), 902–934. DOI: 10.1016/j.jsc.2007.07.002.

they could be much more difficult in one direction than in the opposite direction: integration and differentiation provide a classical example. At a level of undergraduate mathematics, especially in procedure-centered service teaching, it is usually much easier to check the answer than solve an equation. This difficulty could be resolved only by development and introduction of powerful and flexible processors of unconstrained symbolic input of answers to open-ended questions.

We need to understand, however, the unavoidable limitations of CAA: they are better suited for testing routine procedural skills rather than creative thinking and understanding of highly abstract concepts.

We should expect a pressure to switch to CAA not only in formative assessment and coursework tests, but also in course examinations. Indeed, experience shows that a formative CAA translates better to good exam results if the exams are set in the CAA format already familiar to students. There is a danger that if students see that the use of CAA for formative assessment helps to achieve desired test and exam results they are likely to make the CAA their learning tool of choice and ignore other forms of learning.

"Teaching to the test" is already a dangerous but underestimated trend that rapidly erodes the fundamentals of mathematical education. The main danger associated with the CAAs is that their easy availability will increase the already existing pressure to "teach to the test"— and, which could happen to be a much worse outcome—"to teach to the *computerised* test". Paradoxically, the more successful a CAA the more harm it may bring to mathematics education in the long run.

There are further paradoxes. An efficient formative assessment may improve students' performance but depress students' satisfaction with the course. I refer the reader to a cautionary tale told by Mike Fried [[†]]: his ICT solution used in undergraduate teaching of vector calculus

> *"... led students to see they could work harder. Many of my students (certainly not all) interpreted that as a negative."*

Another cautionary tale as documented by Grehan et al. [[‡]] is of more general nature: students' reluctance to get any form of feedback:

> *"students reported that the fear of their own emotional reac-*

[†] M. Fried, Classroom assessment vs. student satisfaction, Notices AMS 58 no. 2 (2011), 229. http://www.ams.org/notices/201102/rtx110200229p.pdf.

[‡] M. Grehan, C. Mac an Bhaird, and A. O'Shea, Why do students not avail themselves of mathematics support? Research in Mathematics Education 13 no. 1 (2011), 79–80. DOI: 10.1080/14794802.2011.55073. Stable URL: http://dx.doi.org/10.1080/14794802.2011.550736.

tion to failure stopped them from seeking help or even from attempting assignments".

This opens yet another can of worms: evaluation of courses and what aspects of learning and teaching are actually evaluated [†]. This theme, however, lies outside of scope of this paper.

11. *ICT based distance Learning*

Mathematics is difficult. It is essential that students of mathematics receive in their learning close personal support from their peers and teachers. Distance learning (including ICT based distance learning) can only operate effectively if this support is not destroyed or displaced.

There is already some experience in the use of ICT based distance learning methods in teach mathematics at university level. A notable recent use of this has been in the EPSRC *Taught Course Centres*, such as, for example, the MAGIC consortium. In these centres, leading research universities work together to deliver a focused programme of graduate level teaching through a video conferencing approach. There are four important features of graduate teaching which make this approach both necessary and viable.

- Firstly, graduate teaching is specialized and is delivered to *small groups*. If such small groups were on their own in a single university then it might be simply inviable in resources to teach them, however, by joining together then the classes across several universities one gains a critical mass and the classes then become resource effective.
- Secondly, the classes are delivered by clusters of universities that work together as a team on an equal footing.
- Thirdly, graduate students by their very nature are able and motivated students.
- Fourthly, all of the students on the course will have extensive back-up and support *at their own universities*.

But, none of these four points above apply to *undergraduate teaching* where classes are large. In a scenario promoted by some politicians [‡] one university would deliver distance learning to a series of much smaller institutions. The main danger here is that at the receiving end of this delivery

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[†] S. Zucker, Evaluation of our courses, Notices AMS 57 no. 7 (2010), 821. `http://www.ams.org/notices/201007/rtx100700821p.pdf`.

[‡] "Universities need radical overhaul, says David Willetts". `http://news.bbc.co.uk/1/hi/education/10278662.stm`.

system we are likely to find a significant number of weakly prepared or unmotivated students and students without direct support at their home base. The experience of the Open University, which has been using distance learning courses for a long time, is that they only work because the students on them are highly motivated and have access to tutors and extensive resource materials.

Appendix at the end of this paper contains assessment (generally positive) of MAGIC courses by five postgraduate students from School of Mathematics of the University of Manchester. On the basis of their personal experiences as students they came to same conclusions as the ones stated above: personally, they found MAGIC courses useful. However, one of the students put his judgement in the following words:

> *I would strongly discourage distance learning becoming a major part of undergraduate education.*

12. *Shopping List*

Mathematicians do not want to work in isolation from the rest of the ICT learning community; there are a number of issues (like support to users with disabilities) that need a coordinated effort.

Here is a brief list of our concrete wishes. It was suggested by my colleagues who read earlier versions of my notes. Any help and advice from the ICT learning community would be warmly appreciated—but not "one size fits all" solutions!

- **Virtual Learning Environments**:
  * Support for, and interfacing with, MATLAB, MATHEMATICA, MAPLE, SPSS, R.
  * Support for symbolic input and output in MATLAB, MATHEMATICA, MAPLE, and import of mathematics graphics produced by these packages.
  * In SPSS and R—input and output of data files, import of tables.
  * As it was already explained before, VLEs are unusable in mathematics learning and teaching if they do not support for TEX / LATEX.
  * One of the benefits cited for VLEs is the ability for students to engage in discussions. They cannot do this if we have barriers to getting mathematics into a machine. Thus support for TEX / LATEXis essential.
- **Computer Aided Assessment**: we need systems that allow easy and

unconstrained entry of mathematical formulae and are able to interpret their meaning.

– **Provision for visually impaired students**. Screen readers do not work with mathematics!
  Visually impaired students need notes in Braille which adequately present mathematical formulae, but they also need ways of interacting with graphical displays on computer screens. In particular, it would be useful to develop LaTeX tools for easy conversion of teaching materials into a format accessible to visually impaired students.
  Some of potentially useful solutions appear to be relatively straightforward from a technical point of view; for example, it appears natural to try to develop a mark-up language for embedding into LaTeX files that would provide creators of LaTeX files with tools for creation and control of PDF tags in output PDF files (thus making tables and footnotes accessible to keystroke navigation) and for writing from LaTeX directly into the accessibility layer, making, for example, mathematical formulae readable by screen readers. One may think about something like

```
\[\int_0^1 2x^3 dx\]%
\readaloud{%
integral from zero to one
of two "x" cube "dx"}
```

  being converted into a pdf file which properly renders LaTeX on the screen, as

$$\int_0^1 2x^3 dx,$$

  while the argument of `\readaloud` is being read aloud (without, of course, being shown on the screen).
  One immediate difficulty is that there are no even universally accepted rules for reading complex mathematical formulae aloud.

– And last but not least—the role and status of free and / or **open source software, courseware and textbooks** deserve a thorough discussion.

13. *Acknowledgements*

I wish to express my thanks to my colleagues from the Education Committee of the London Mathematical Society and from the Research Committee of the Association for Learning Technology [†] for very useful discussions.

† `http://www.alt.ac.uk/`.

My special thanks go to Chris Budd, Scott Carter, Burçin Eröcal, Dick Hudson, Stephen Huggett, David Mond, Morag Munro, Chris Sangwin, Seb Schmoller, Christopher Stephenson, Brian Stewart, and to a group of PhD students in School of Mathematics of the University of Manchester who preferred to stay anonymous.

Earlier and much shorter versions of this paper have been published in *ALT News Online* [†] (in a preliminary form) and in *Research in Learning Technology* [‡]; I am grateful to the anonymous reviewers of the journal version. It is worth mentioning that *Research in Learning Technology* becomes a fully Open Access journal with effect from 1st January 2012 [§].

Some parts of this paper were completed during my visit to the Nesin Mathematics Village [¶]. My thanks go to Ali Nesin and all staff, volunteers and students at the Village who created for me a fantastic work environment.

14. *Appendix: Students' assessment of* MAGIC *courses*

I include anonymous responses from 5 postgraduate students at Manchester University who take course delvered via MAGIC programme.

14.1. MAGIC*: Response 1*

I would say that the 'distance learning' model has the potential to be a very effective way of delivering lectures to large audiences, however, a couple of major changes would be required to the way lectures are currently delivered in MAGIC courses.

The main issue I have is that I think that slides are a very ineffective way to teach mathematics. It is much easier to follow a lecture if the lecturer writes out the notes as he/she goes along. The best way I have seen this achieved in MAGIC courses is with the use of a visualiser.

It would also, of course, be essential to make sure that the technology worked 100% of the time. There have been technical issues at a large num-

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[†] *Association for Learning Technology Online Newsletter* 20 (11 August 2010). ISSN 1748–3603. `http://newsletter.alt.ac.uk/4edkkzb138s`.

[‡] A. Borovik, Information technology in university-level mathematics teaching and learning: a mathematician's point of view, Research in Learning Technology Vol. 19, No. 1, March 2011, 73–85. ISSN 0968–7769 print/ISSN 1741–1629 online.
DOI: 10.1080/09687769.2010.548504

[§] S. Schmoller, Journal tendering for societies: A brief guide. 04 Apr 2011. `http://repository.alt.ac.uk/887/`.

[¶] `http://matematikkoyu.org/en`.

ber of the MAGIC lectures I have attended this year, mainly involving sound quality.

If these two points were addressed, it is my opinion that the “distance learning” model could be an effective way to deliver lectures at both post-graduate and undergraduate levels.

14.2. MAGIC*: Response 2*

I can give some quick comments on how I felt the video conferencing worked for some of the MAGIC courses I attended:

Probably my main thought is that it is the case that the system is not quite solving the right problem for teaching maths (and possibly other subjects). Really what I think is needed is just the ability to send sounds and a fairly good resolution image of a black or white board and obviously the ability for using pdf for slides.

What I found was the electronic whiteboard unnecessarily complicates matters and was quite unreliable and difficult to read. Most people ended up just trying to make out what was written by looking at the quite low resolution blurry image that was transmitted. So just sending a slightly higher resolution image would have been enough and you would save the money of buying the whiteboard!

I think the good points are that you can get access to courses that you otherwise would not have the chance to attend. But for undergraduate courses I think there is the obvious big problem that you cannot do problem solving/examples classes that way. Also the reliability of the technology is still an issue, often we had problems with sound and the picture cutting out etc. Even if you could transmit a sufficiently high resolution image of a black/whiteboard and the sound, where will people watch it? If it is at a similar lecture theatre with the microphones/suitable high speed internet connection and cameras it would be ok, but what if the student is expected to do it from their own computer at home. They would then need a camera etc and a reliable web connection with enough bandwidth, all of which complicates things.

For PG education I think it has a place since examples classes are less common but I do not think you could do the majority of UG classes that way unless a solution to videoconferencing examples classes is thought of.

14.3. MAGIC*: Response 3*

The MAGIC courses are good because they allow us to follow a wider range of postgraduate courses than we would be able to as local courses. This far out weighs the negative points in the context of postgraduate education.

Problems with distance learning for undergraduate courses:

- Lack of contact with other students taking the course. This would have a negative impact in two ways, firstly this would make it difficult for students to help each other with courses. For the first two years as an undergraduate I had a friend who I tried to teach analysis and he tried to teach me the more physics based courses. This was incredibly useful. In my experience maths student also helped physics, economics and engineering students with their maths. In later years they also talked to philosophy students taking philosophy of maths and philosophy of physics courses.
  Secondly, students need to be able to talk to other students taking the course, otherwise it can be very isolating. I imagine distance learning full time would greatly increase the number of people ho drop out which I guess in turn would lead to the lowering of standards/content.
- It is possible to talk to the person lecturing through magic but it feels very impersonal. I think it would make lecturers appear even more "unapproachable" than they may currently seem to a lot of undergraduates.
- In my experience of distance courses, they are not very interactive. Local lectures are also not very interactive but at least in example classes and supervision students (should) be playing an active part in the class. I can't see how this would ever be possible with distance learning.
- The technology breaks down to some extent relatively regularly which disrupts the lecture. This rarely happens with chalk and a blackboard.

Finally, I can't imagine the majority of our first year undergraduates having the maturity to study alone to the extent that distance learning would require (which I guess is a fault of the school system but I don't see it changing anytime soon). I know distance learning works for OU but their students are generally significantly older and not taking these courses full time.

14.4. MAGIC*: Response 4*

I will first highlight the good points about videoconferencing technology.

It is extremely useful for general audience lectures/seminars, or one-off events. It is also useful for collaborators who in normal circumstances are unable to talk face-to-face about their work or a paper they are writing. I know of some academic staff who have indeed used the MAGIC room to communicate and discuss a paper they were both writing, and they said it was more productive than e-mail or phone. On the whole, the technology is a good invention and has many practical applications.

However, I do not believe that the 'distance learning' model is a good application. For one, there is never any substitute for a living breathing person in the lecture room conveying material. My experience with MAGIC (and indeed the general consensus) is that watching somebody on a screen is by far inferior. I would not have been happy if any portion of my undergraduate degree had been via videoconferencing technology.

Another issue is that, even though a lecturer may be lecturing to hundreds of people, if somebody wants to ask a question mid-lecture they can do and the lecturer can make eye-contact with this person and relate the person to the question being asked. My own experience tells me that it is awkward for both the lecturer and the student to communicate in this way when they are mid-lecture.

Finally, there will always be technology problems. Even 3 years on, the MAGIC system is not flawless and I will never believe it will be. For somebody using "distance learning", if the technology fails then that person has missed out on any form of lecturer contact for that portion of the course. If a lecturer is taken ill suddenly, then everybody is affected and the course can globally pause and everybody is no worse off. If the technology fails in one centre, however, the lecturer may not know this and that centre is then one lecture behind everybody else.

All in all, my feelings are: The technology is a fantastic invention and does have a place in the university, but only in the form of collaboration, or one-off events. But if it were my choice, I would keep it away from a regular teaching model.

14.5. MAGIC*: Response 5*

I have sat several MAGIC courses over this past academic year and I found them very useful for my research. However, the technology itself has been somewhat troublesome at times. Technical problems like poor sound, faulty mike, poor visuals etc. have been so severe at times that lectures have actually had to be cancelled. Even when they were not cancelled the problems made it difficult to follow the lectures.

In spite of this, in general I think that the technology is an invaluable tool for postgraduates as it offers a good selection of advanced level courses that one would not normally be able to get in one's own institute. Having said this, I do not think that distance learning in anyway replaces on-site learning for the following two reasons-

- It is far more uncomfortable to ask a question if you are not in the same site as the lecturer.
- The lecturers cannot see the people at different sites so cannot gauge whether they are understanding the material or not.

Unless these two problems can be remedied **I would strongly discourage distance learning becoming a major part of undergraduate education**.

Disclaimer

The views expressed do not necessarily represent the position of my employer or any other person, organisation, or institution.

About the Author

I am an research mathematician but I have 40+ years of teaching experience at secondary school and university level in four different countries with four different education systems; since 1998 I am a Professor of Pure Mathematics at the University of Manchester.

I also have an interest in cognitive aspects of mathematical practice; see my book *Mathematics under the Microscope* [†], which explains a mathematician's outlook at psycho-physiological and cognitive issues in mathematics and mathematics education, and touches on many issues raised in this paper. Some of my papers on mathematics education can be found in my personal online journal/blog *Selected Passages From Correspondence With Friends* [‡].

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† A. V. Borovik, *Mathematics under the Microscope: Notes on Cognitive Aspects of Mathematical Practice*. Amer. Math. Soc., Providence, RI. 317 pp. ISBN-10: 0-8218-4761-9. ISBN-13: 978-0-8218-4761-9. Available from http://www.ams.org/bookstore-getitem/item=mbk-71.

‡ Selected Passages From Correspondence With Friends. ISSN 2054–7145. http://www.borovik.net/selecta/.