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SUBLIME SYMMETRY: MATHEMATICS AND ART

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Introduction

This paper is a text of my talk at the opening of the Exhibition *Sublime Symmetry: The Mathematics behind De Morgan's Ceramic Designs*† in the delighful Towneley Hall‡, Burnley, on 5 March 2016. The Exhibition is the first one in *Sublime Symmetry Tour*§ organised by The De Morgan Foundation¶.

I use this opportunity to bring *Sublime Symmetry Tour* to the attention of the British mathematics community, and list Tour venues:

– 06 March to 05 June 2016 at Towneley Hall, Burnley
– 11 June to 04 September 2016 at Cannon Hall, Barnsley
– 10 September to 04 December 2016 at Torre Abbey, Torbay
– 10 December 2016 to 04 March 2017 at the New Walk Gallery, Leicester
– 12 March to 03 September 2017 at the William Morris Gallery, Walthamstow

My talk at the event was improvised; I wrote it down next day and took the liberty to add some relevant illustrations.

My talk

Opening statement. Thank you to everyone who was involved in, or contributed to, setting up this magnificent exhibition. My special thanks go to Sarah Hardy, the Exhibition Curator, for an informative, analytic, and deep chapter in the Exhibition Catalogue. Her work is a proof that curating is an art form on its own.

I also wish to say cordial thanks to Sarah Hardy and Claire Longworth from the De Morgan Foundation who came up with a wonderful title for the Exhibition: *Sublime Symmetry*. It is a mathematical message: frequently the symmetry properties of an object, structure, or artefact, are not immediately obvious – but affect their deeper meaning and intrinsic value.

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†*Sublime Symmetry: The Mathematics behind De Morgan's Ceramic Designs*, `http://www.demorgan.org.uk/loans-and-exhibitions/sublime-symmetry`.

‡Towneley Hall, `http://www.burnley.gov.uk/residents/towneley-hall`.

§*Sublime Symmetry Tour*, `http://www.demorgan.org.uk/loans-and-exhibitions/sublime-symmetry`.

¶The De Morgan Foundation, `http://www.demorgan.org.uk/index.php`.

It is a privilege for me to represent the London Mathematical Society[‖] at this event. Last year, the Society celebrated its 150th Anniversary; it was founded in 1865 by Augustus De Morgan, father of William De Morgan.

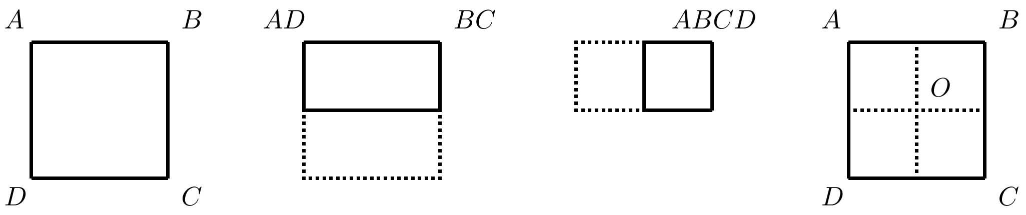
Augustus De Morgan obviously influenced his son's world view, mathematical in its nature. This is why I will try to to explain the roles of symmetry in mathematics and in the Art – as we understand them now.

The most basic rule for public talks about mathematics is *be brief*. I promise to stick to it and never mention a single mathematical formula.

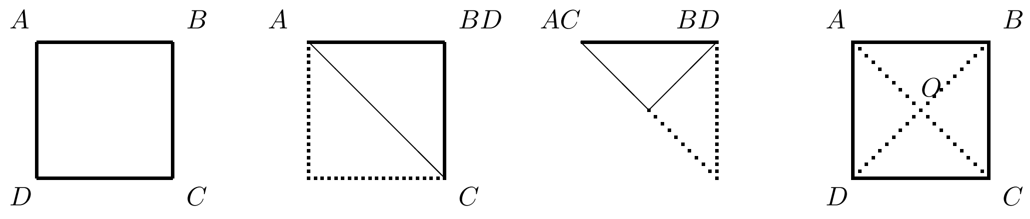
S*ymmetry in mathematics.* Mathematics is the art of precision; it deals with mental constructions and procedures which can be reproduced with arbitrary (you may wish to say, absolute) precision. A theorem proved by Euclid (and William De Morgan was an admirer of Euclid) is just as true and valid now, two millennia after Euclid's time. Transfer of mathematical knowledge from generation to generation has never resulted in schisms of the kind that triggered one religious war after another – and these wars continue to burn right now, as we speak here.

I'll show you now a simple example of high precision reproduction of a mathematical object. As you see, I hold two paper napkins, and I'll fold them for you in two different ways.[†]

I fold the first napkin that way[‡]:



and another napkin that way:



As you can see, in the both napkins the point O, where the two creases intersect, is the same point, the centre of the square. And this is a mathematical fact: it was

[‖]London Mathematical Society, `www.lms.ac.uk`.

[†]I have actually showed this simple experiment to the audience, using two paper napkins borrowed from the table with delicious homemade cookies provided by Towneley Hall Society, `http://towneleyhallsociety.co.uk/`, volunteers for this particular occasion.

[‡]I borrowed this example from my book *Mathematics under he Microscope: Notes on Cognitive Aspects of Mathematical Practice*, American Mathematical Society, Providence, R.I., 2010, p. 226.

true 2,000 years ago, and it is true now. Please notice also that the resulting patterns of creases are very symmetric – moreover, they have well established symbolic meanings: for example, in the second napkin it is the distinctive St. Andrew's cross.

And now I can formulate the main thesis of my talk:

> *Symmetry of an object is a proof of precision of the process that led to its creation.*

This is almost a banality; but this is a really important role of what I call "*static*" symmetry: it acts as a certificate of precision of reproduction. We shall soon see that this principle equally applies in Mathematics, in Nature, and in Art.

But mathematics also has the concept of symmetry as a *dynamic* process: symmetry is a reversible transformation of objects and structures which preserve some of their properties. The word "reversible" is important here; it means that we can always *check* and *correct* the result of transformation.

Instead of further examples and explanations, I offer you a classical conundrum which deceptively appears to capture the essence of the problem but instead demonstrates that, frequently, symmetry is not what it appears to be:

> *Why does the mirror change left and right but does not change up and down?*

FIGURE 1. *Peacock Dish.* William De Morgan. The De Morgan Foundation, used with permission.

Symmetry in Nature – and in Art. Let us have a look at one of the most elaborate objects of beauty in Nature: a peacock's tail. It has attracted the attention of artists over millennia, and was noticed by William De Morgan, see Figure 1.

The wonderful ornament of a peacock's tail makes sense only if it is appreciated by peahens. What kind of message does it send to a peahen? That a high precision, high symmetry ornament of a feather (the famous "*eye*") formed by thousands of individually colored hairs independently growing from the feather's stem (see Figures 2 and 3) could be produced only by an exceptionally fine tuned copying mechanism. And this mechanism is encoded in a peacock's genes. Hence a peacock with an elaborate tail can be a good biological father of peahen's chicks – her own genes will benefit from high precision copying.

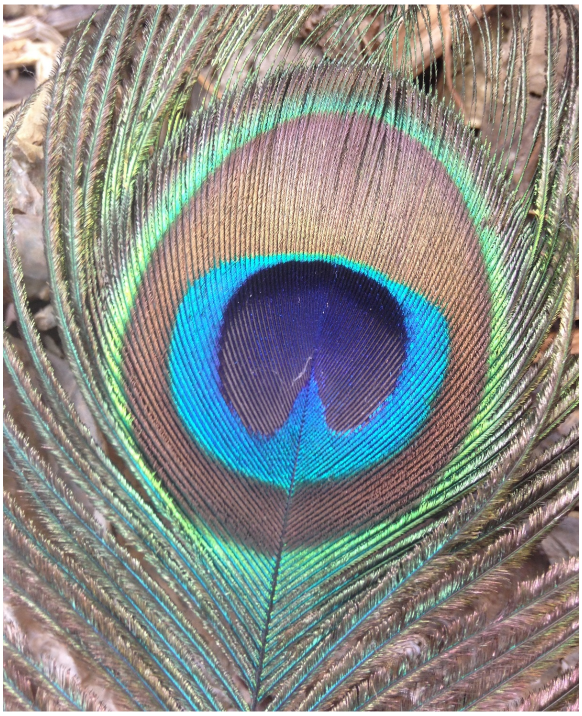


FIGURE 2. *Close up of a peacock feather.* WIKIMEDIA COMMONS. Public domain.

Why are peacocks' tails attractive to humans? Because in humans as a species, biological beauty is a marker for healthy genes. A beautiful face is, first of all, a bilaterally symmetric face. A beautiful body is (almost) bilaterally symmetric. And, not surprisingly, beauty as a biological stimulus is the basis of visual art, bringing with it artists' and spectators' attention to (and maybe even obsession with) symmetry.

Two archetypes. And now I would like to mention two archetypal Greek myths which involve mirrors and reflections – that is, symmetry.

Perseus and Medusa. To give a very brief retelling of a Greek myth, Medusa had poisonous snakes on her head and a petrifying gaze which turned into stones

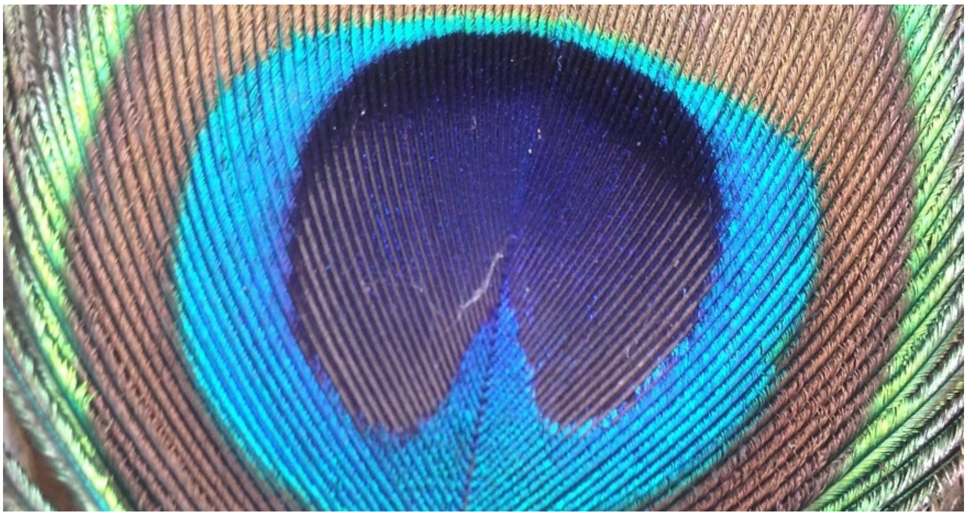


FIGURE 3. *Peacock feather under higher magnification.* WIKIMEDIA COMMONS. Public domain.

FIGURE 4. *Perseus beheads Medusa.* Luigi Ademollo (1764–1849). *Illustration from ovid's Metamorphoses, Book IV. Florence, 1832.* WIKIMEDIA COMMONS. Public domain.

people who dared to look into her face. When Perseus was given an order to kill her, Athena provided the hero with a polished bronze shield which could be used as a mirror. Using it, Perseus killed Medusa by beheading her – and without looking at her directly, Figure 4. He successfully used the severed head as a weapon: the horrible gaze remained lethal.

De Morgan's contemporary, the Pre-Raphaelite artist Edward Burne-Jones, gave a more delicate treatment to the same theme: in his painting *The Baleful Head,*

Figure 5. *The Baleful Head, 1887.* Edward Burne-Jones (1833–1898). Wikimedia Commons. Public domain.

Perseus shows the reflection of Medusa's head – not the head itself – to his beloved Andromeda, Figure 5.

The mirror reflected Medusa's shape and position, thus helping Perseus to aim his sword, but did not reflect the paralising horror that she radiated. Perhaps, water in the well is seen as even better protection. Compare that with astronomy: it is dangerous to look directly at the Sun, but a simple pinhole projection of the Sun onto a wall in a dark room (and projection is a form of symmetry) allows one to see sunspots in complete safety.[†]

[†]See, for example, *Pinhole Astrophotography*, `http://users.erols.com/njastro/barry/pages/pinhole.htm`, and do not miss a striking observation: "*the large sunspots, as well as those of moderate size, could easily have been viewed using pinhole projection by some ancient observers had they been so inclined.*".

This is similar to the use of symmetry characteristic for mathematics: the instrumental use, use as a tool, as a weapon, as a filter for properties which we wish to preserve in the image, while changing the others.

FIGURE 6. *Narcissus.* Caravaggio, 1594–1596. WIKIMEDIA COMMONS. Public domain.

Narcissus. This one is a very different story – Narcissus (allegedly, as a punishment from the gods) fell in love with his reflection in the lake, Figure 6.

And this use of symmetry is typical for art, and transcends decorative art:

the mirror can reflect the soul.

Even in the decorative art,

symmetry can be a medium for emotions,

and the latter is fairly obvious in works by Willian De Morgan.

Conclusion. Summarising the myths and the stories, I can claim that

Symmetry in art is the human face of mathematics.

William De Morgan's art shows that mathematics is human. From the bottom of my heart – thanks to everyone who helps to spread this message.

Thank you.

Acknowledgements

This paper would have never been written without the kind invitation from The De Morgan Foundation and without support from the London Mathematical Society.

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I am grateful to Stephen Huggett and Tony Gardiner who suggested useful corrections in my text.

My special thanks go to staff of Towneley Hall (and Towneley Hall and its art collection are treasures on their own) and to members of the Towneley Hall Society for their warm hospitality.

The De Morgan Foundation kindly allowed me to use the image of *Peacock Dish* by William De Morgan, Figure 1, and provided a high resolution image file.

Disclaimer

The author writes in his personal capacity; his views do not necessarily represent the position of his employer or any other person, corporation, organisation or institution.

About the Author

I am a mathematician. I study symmetries (and have even co-authored a textbook *Mirrors and Reflections*[†]). More precisely, I study multi-mirror reflections, where mirrors get reflected in other mirrors, and reflections breed and multiply all over the place. I design mathematical methods for finding safe paths in the maze of mirrors, and for telling real objects from sneering phantoms.

If you expect to get involved in a shoot-out in a Hall of Mirrors (like the one in the famous scene from *The Lady from Shanghai*[‡]), Figure 7, I am happy to provide some advice.

[†] A. V. Borovik and A. Borovik, *Mirrors and Reflections. The Geometry of Finite Reflection Groups.* Springer, 2010.

[‡] *The Lady from Shanghai*, Orson Welles, 1947. See a YouTube clip of the Hall of Mirrors sequence at `https://www.youtube.com/watch?v=_RdPVtcDeEI`.

FIGURE 7. The mirror scene from *The Lady from Shanghai.* WIKIMEDIA COMMONS.