Selected Passages from Correspondence with Friends 3 no. 2 (2015), 7–8. ISSN 2054–7145

DERIVATION IN A LEVEL MATHEMATICS

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This a comment on the post[†] at *mathematicsandcoding*:

> I recently asked my Year 12 FP1 students to derive an expression for the inverse of a general 2×2 matrix, in terms of the entries of the matrix that you are trying to invert. Of course, a straight forward way to do this is to use the definition of the matrix inverse to derive 4 equations with 4 unknowns which can be solved in two pairs to obtain the entries of the inverse matrix in terms of the original matrix.
> I was surprised to find that no one managed to do it. Perhaps the amount of letters floating around put them off, but I think this kind of exercise is important for building confidence and developing fluency with methods used in other modules.
> I typed up a solution for them, if it is of interest to anyone it is available on my website here[‡].

In my opinion, a good problem on derivations is the one where a solution that can be realistically found by students is reasonably good why should we push them towards bad derivations? Good derivations frequently (but not always) involve use of connections between various aspects of mathematical objects and concepts involved.

Regarding the example referred to in *mathematicsandcoding*, derivation of the formula for the inverse of a 2×2 matrix by brute force—by solving a system of 4 equations with 4 variables, is an example of not so good derivation. Indeed, this can be done much simpler.

Given a matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

we want to find a matrix

$$\begin{bmatrix} x & y \\ z & w \end{bmatrix}$$

2010 *Mathematics Subject Classification* 97H60.
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[†]Why are there so few derivations in A Level Mathematics? `http://blog.ifem.co.uk/why-are-there-so-few-derivations-in-a-level-mathematics/`. Accessed 17 February 2015.

[‡]The inverse of a general 2×2 matrix.`http://ifem.co.uk/files/inverse_2_by_2_matrix.pdf`. Accessed 17 February 2015.

such that

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

the equation for 0 in the upper right corner is

$$ay + bw = 0,$$

which has a solution $y = -b$ and $w = a$ up to multiplication by a constant, say λ. Similarly, for the bottom left corner we have $x = d$ and $z = -c$, again up to multiplication by a constant, say μ. To find λ and μ, we multiply the matrices and discover that

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} ad - bc & 0 \\ 0 & ad - bc \end{bmatrix}.$$

Now we can forget about λ and μ because we already see that

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

We found *an inverse*; but I hope that children already know that the inverse is unique.

I think children may benefit more from being shown this derivation rather than forced to solve a system of 4 equations with 4 variables.

After all, mathematics is the art of avoiding computations.

Disclaimer

The author writes in his personal capacity and the views expressed do not necessarily represent position of his employer or any other person, organisation or institution.

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