## EXPRESSION OF MATHEMATICS IN HUMAN LANGUAGES: A QUESTIONNAIRE

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### 1. A few words of explanation

For a number of years, I have been teaching a course in basic mathematics for Foundation Studies students. This year, the course had 340 including about 70 students from overseas. The aim of the course is to prepare students who did not study mathematics in the last two years of secondary school, or who came from overseas, for harder mathematics courses or mathematically intensive engineering courses that they are likely to take in their later university studies.

The course covers some elementary set theory and propositional logic, a bit of number theory, a bit of polynomials—not a systematic theory, but a working minimum of material for learning *the language of mathematics*.

Students in the course come from the bewildering variety of socioeconomic, cultural, educational and linguistic backgrounds. Just at the level of basic notation, I have to deal with students who, in their school mathematics, were using two different symbols for multiplication:

$$2 \cdot 3 = 6$$
 and  $2 \times 3 = 6$ ,

and three different symbols for division:

$$6/3 = 2;$$
  $6: 3 = 2;$   $6 \div 3 = 2.$ 

Some countries use decimal point:

$$\pi = 3.1415\ldots,$$

while others prefer decimal comma:

$$\pi = 3,1415...$$

- and this list can run on and on.

Much more obstructive are invisible differences in the logical structure of my students' mother tongues. For example, the connective "or" is strictly exclusive in Chinese: "one or another but not both", while in English "or" is mostly inclusive: "one or another or perhaps both". Meanwhile, in mathematics "or" is always inclusive and corresponds to the expression "and/or" of the bureaucratic slang.

In another example, the so-called *distributive numerals* (absent in English) make Turkish grammar biased towards expression of exact division, without leftovers,

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and make it more difficult to master division with remainder which so prominently features in the course.<sup>†</sup>

The principle of *material implication*:

IT IS TRUE THAT "FALSE" IMPLIES "TRUE"

is unacceptable and even offensive to many students for deeply rooted cultural reasons.

Again, this list can be continued indefinitely.

# 2. Questionnaire

The aim of the following questionnaire was to gather basic information about those features of various languages which significantly affect expression of mathematical concepts and arguments.

I would be most grateful to the readers if they could provide me with answers (or expand / correct existing answers) in respect of their mother tongues.

- (1) Your answers to this questionnaire—about which language are they? (In my questions, I refer to this language as 'your language').
- (2) In your language, are there (a) definite (b) indefinite articles? (English has both, Russian—my mother tongue—neither).
- (3) In your language, is there a distinction between singular, dual (do not be surprised, some languages have dual forms) and plural forms of (a) nouns; (b) verbs? (Russian has singular and plural, both for verbs and nouns, with an additional curious occurrence of plurality markers for two, three, four objects (nouns), which are different for plurality markers for 5+ objects.)
- (4) Is 'double negation' used in normative (literate) speech?

In English this is usually a sign of some form of slang: 'I do not have no knife, man!' In Russian, double negation is used to strengthen the denial: Y meня нет никакого ножа, with two connectives  $\mu e$  and  $\mu u$  used as a primary negation ( $\mu e$ ,  $\mu em$ ) and its re-enforcement ( $\mu u$ ).

- (5) Is the verb 'to be' and its forms routinely omitted from sentences? (This is a prominent feature of Russian.)
- (6) Connective 'or'—is it usually understood in inclusive (and/or) or exclusive (either one, or another, but not both) sense? Or can its meaning vary depending on context? In English and Russian, it is mostly inclusive, with an occasional

<sup>&</sup>lt;sup>†</sup>Distributive numerals are discussed in some detail in my draft book *Shadows of the Truth: Metamathematics of Elementary Mathematics*, http://www.maths.manchester.ac.uk/avb/ST.pdf.

slip into the exclusive mode: 'choose: her or me!', *я или она!* 

- (7) I was told that the Japanese language makes use of two different connectives 'and': one for nouns, another for verbs. In your language, are there situations where English connectives 'and', 'or', 'but', 'not', 'if ... then' have to be translated in two or more different ways depending on context?
- (8) Do you know any traditional proverbial saying which expresses the logical tautology

$$p \to (q \to p)?$$

I discovered that, apparently, students coming from some cultural backgrounds are somewhat uncomfortable with the principle of material implication (which is, in effect, expressed by the tautology above). Russians have such proverbs, for example

 $Tyз-он u \ e \ A \phi pu \kappa e \ mys$  ('Ace is the Ace, even in Africa').

(9) Any other observations about the logic of your language and peculiarities of its use in mathematics.

In your responses, please do not hesitate to use MS Word, DVI or PDF, if this is necessary for capturing the peculiarities of the alphabet / script of your language or email me a handwritten graphics file (I could have difficulty with hieroglyphs, though!).

Please send your answers and comments to

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What follows are two examples from many responses that I have already received from my colleagues.

# 3. A sample response: A letter from David Pierce about Turkish language

Let me address Turkish as I understand it (from my experience, from talking with Ayşe, and from reading Geoffrey Lewis<sup>†</sup>).

- (1) Language: Turkish.
- (2) Articles. Short answer: there is an optional indefinite article, but no definite article. Longer answer: The word *bir* 'one' can serve as an indefinite article. Used like 'one', it precedes adjectives; used like 'a(n)', it follows them: *bir güzel gün* 'one fine day' *güzel bir gün* 'a fine day'

But the indefinite article is not required. Its omission may express even more indefiniteness:

Her hafta kitap okuyorum 'Every week I read a book' (that is, every week I spend time reading a book).

*Her hafta bir kitap okuyorum* 'I read [and finish] a book every week.'

Moreover, there are two cases for direct objects. An indefinite direct object takes the neutral form, like a subject; a definite direct object carries an ending. But a definite direct object may still take the indefinite article:

Her gün bir gazete okuyorum 'Every day I read a newspaper' (not necessarily the same one each day).

Her gün bir gazeteyi okuyorum 'Every day I read a newspaper' (a particular newspaper, such as *Cumhuriyet*).

The 3rd-person pronoun o 'he/she/it' serves also as a demonstrative adjective:  $o \ kitap$  'that book'.

But o is not used as a definite article:

*Kitap masada* 'A book is on the table' or 'The book is on the table.'

(3) Number.

(i) Nouns. The numerically neutral form of a noun can be made into an indefinite plural by means of a suffix (-lar or -ler). The suffix is not used if a word for a specific number is used: başlar 'heads'; beş baş 'five head'. There is at least one example of an Arabic word whose singular and plural forms exist as separate words in Turkish:

sey 'thing' (hence seyler 'things');

eşya (or eşyalar!) 'things, belongings'.

Geoffrey Lewis says a few Arabic dual forms survive barely in Turkish: ebeveyn '[one's] parents'; taraf 'side', tarafeyn 'the two sides [parties]'.

(ii) Verbs. In the first and second persons, there are distinct endings for singular and plural. (I mean for example that a first-person plural ending cannot be analysed into a first-person part and a pluralizing part, though this may have been possible historically.) The second-person plural form is also used as a respectful singular.

There is no third-person ending for verbs;—rather, the third-person ending is empty (except in the imperative). However, in case of a plural (third-person) subject, the pluralizing ending -lar/ler is used on the verb (optionally in case of an inanimate subject).

 (4) Double negation. The pronoun hiç 'never/nothing' can emphasize negations: *Gerek yok* 'There is no need' *Hiç gerek yok* 'There is no need at all'

Perhaps a true 'double negation' is shown in the following:

Görmedim 'I didn't see [anything].' Hiç görmedim 'I saw nothing at all' (cf. 'I din't see nuthin'.')

- (5) 'Be': Routinely omitted. See 'my' article Turkish copula in Wikipedia for six ways of ... copulating in Turkish.
- (6) Disjunction. For most disjunctions, which may be practically exclusive, but where exclusivity is not the point, there are veya and ya da; where exclusivity is emphasized, there is ya... ya. There is also yoksa 'if not': Erkek veya kadin, fark etmez 'Man or woman, it doesn't matter.' Ya sev, ya terk 'Love it or leave it.'

Cay isterim, yoksa kahve 'I want tea, or, if there is none, coffee.'

(7) Conjunction. The Arabic borrowing ve 'and' is an all-purpose conjunction, usable with nouns and verbs, but it is not spoken much. From its own resources, Turkish has different ways of conjoining nouns and verbs. Nouns can be conjoined with the postposition *ile* 'with':

Ahmet ile Mehmet 'Ahmet and Mehmet' (lit. 'Mehmet with Ahmet'). When two adjacent verbs have the same subject and the same endings, then the ending of the first can be replaced with -ip to denote conjunction: Kalkip gittik 'We got up and left' (in place of Kalktik ve gittik).

- (8) Material implication. I have no idea.
- (9) In English, the statement
  - (i) Everything is A

can be formally negated in two ways:

- (ii) Something is not A;
- (iii) Not everything is A.

But perhaps many people will still write or especially speak the negation as

(iv) Everything is not A.

(Compare the saying, 'All that glitters is not gold.') However, if we are trying to use the language logically, then we prefer to understand (iv) as having the same meaning as

(v) Nothing is A,

which is a negation of

(vi) Something is A.

In Turkish, sentence (iv) is a negation of (i) rather than of (vi), because there is no possibility of writing (iii):

Her muz Asal muz değildir lit. 'Every banana Prime banana notis', that is, 'Not every banana is a Prime banana' (slogan of a banana company in Antalya).

Her muz Asal muzdur 'Every banana is a Prime banana.'

*Hiçbir muz Asal muz değildir* 'No [i.e. not-one] banana is a Prime banana.'

# 4. One more response: Vedran Čačić about Croatian language

(1) Your answers to this questionnaire—about which language are they?

Croatian. I'll use c = e form for speaking about words and phrases, where c is Croatian, and e English 'equivalent'.

(2) In your language, are there (a) definite (b) indefinite articles?

No articles. There are many words that can 'serve' as indefinite articles:

neki = some,

for example, but no particularly suitable word to serve as definite article (except adjective

određeni = definite,

I guess). Many times when speaking mathematics, I (in desperation) use English articles to convey meaning, e.g.

Misliš da si našao a metodu ili the metodu za rješavanje problema tog tipa? = You mean you found **a** method or **the** method for solving problems of that type?

Though, I haven't seen many other people do that.

(3) In your language, is there a distinction between singular, dual (do not be surprised, some languages have dual forms) and plural forms of (a) nouns; (b) verbs?

Croatian doesn't have dual forms, except in some archaic forms, though some nouns for body parts of which people usually have two, have irregular plurals, probably influenced by dual forms of ancient times:

 $oko = eye, \ o\check{c}i = eyes, \ uho = ear, \ u\check{s}i = ears.$ 

It has singular and plural forms for nouns, verbs, and even adjectives and pronouns

ja = I, mi = we, ti = you singular, vi = you plural. It also has plurality markers for 2, 3, 4 objects different than those for 5,... objects for some nouns:

 $2 \, djeteta = 2 \, children, 5 \, djece = 5 \, children,$ 

but I suppose fewer such cases than Russian.

(4) Is 'double negation' used in normative (literate) speech?

In Croatian, double negation is pretty normal, and in fact I'd say not only double, but 'universal negation', where every relevant part of speech is negated:

Nikada ni na koji način nismo htjeli ostaviti nikakav dojam površnosti = We never in any way wanted to leave any impression of superficiality

-you can count 4 negations in Croatian sentence, it's all words

beginning with ni.

- (5) Is the verb 'to be' and its forms routinely omitted from sentences?
  - Yes, but not routinely, and it depends on word order and the tense of the verb, mostly to avoid clustering of small words:
    - $On \ se \ okrenuo =$  He turned around,

but

Okrenuo se je on —same sentence with another word order, je is auxiliary 'to be' form here.

(6) Connective 'or'—is it usually understood in inclusive (and/or) or exclusive (either one, or another, but not both) sense? Or can its meaning vary depending on context?

Mostly inclusive, but it is really dependent on the context (I would say we need inclusive or more often than exclusive). Usually

 $A \ ili \ B = 'A \ or \ B' \ inclusively,$ 

but

ili A ili B = 'A or B' exclusively.

Same as with articles, sometimes in desperation I use 'xor' in speech when I want to convey the meaning of exclusive or.

(7) In your language, are there situations where English connectives 'and', 'or', 'but', 'not', 'if ... then' have to be translated in two or more different ways depending on context?

Well, Croatian has many connectives, but they are mostly synonymous, so it is just a matter of style which one you'll use. However, there are subtle differences even in many 'synonymous' cases. For example,

i =and and te =and

are synonymous when enumerating objects, but with actions, i usually mean somebody did them in parallel, and te means they did them one after the other.

One thing that's really worth mentioning is a, a connection that is usually translated as 'and', but really is in category with opposing connectives, like 'but'. For example,

Ja radim a ti pjevaš = I'm working and you're singing —means our actions are opposed, although it is obviously the same meaning as *i*. It usually comes up in teaching mathematics when you explain logical connectives, and then you must say to Croatian students that conjunction (&) is not necessarily *i*, but can be *a* or some other connectives depending on context.

(8) Do you know any traditional proverbial saying which expresses the logical tautology  $p \to (q \to p)$ ?

Hmm... I can't remember anything like that, but I'm not very

strong with Croatian proverbs.

(9) Any other observations about the logic of your language and peculiarities of its use in mathematics.

There is one thing that usually pops up every time in elementary logic (besides that phenomenon with conjunction), when you explain quantifiers, and there is a very big problem when you try to explain 'in ordinary language' why the order of quantifiers in front of a formula matters (for all exists is not the same as exists for all), because Croatian has very free word order, and how ever you translate those quantifiers and permute them in language, it always means the same damn thing (the weaker one, to add the insult to the injury).

So you really have to pull out a concrete counterexample (every number has a successor, but there is no 'the' successor of every single number—definite article would obviously help here, but as you remember, Croatian does not have that either).

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URL http://www.borovik.net/selecta/

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