

RELATIONALITY OF TEACHING, THE KHAN ACADEMY AND WORD PROBLEMS

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1. *A brief introduction*

This is the first of my several papers on “word problems” in mathematics education. Most passages in the paper come from letters written in 2011–12, mostly in the context of discussions around the National Curriculum reform in England. They might appear to be outdated, especially because of the fast development of new educational technologies, but, I believe, they continue to be relevant. After all, as Stanislas Dehaene quipped in his book *The Number Sense*[†],

We have to do mathematics using the brain which evolved 30 000 years ago for survival in the African savanna.

For that reason, I believe, discourse on mathematics education should involve historic retrospection on timescale longer than a couple of years.

In my next paper, I will be discussing Igor Arnold’s (1900–1948) paper of 1946 *Principles of selection and composition of arithmetic problems*[‡]—because it sheds much needed light on the current controversy around “modelling”. But in this paper, I will be analysing in depth just one word problem from the Khan Academy’s website[§].

I would like to avoid making an impression that I criticise the Khan Academy—it is perhaps one of the best among many on-line projects in mathematics education.

I do not make any judgement of the Khan Academy as a whole. Moreover, I believe it is wrong to judge a systematically developed learning material after looking at the very first randomly chosen exercise. I would not write a similar post about a random exercise in a textbook (I mean, a real textbook, printed on paper). Almost by default, textbooks are linearly structured. On the contrary, the highly modularised structure of the Academy’s website makes it open to assessment of randomly chosen nodes in its topics flowchart (“Knowledge map”).

By the way, let us record this simple methodological observation for future reference:

2010 *Mathematics Subject Classification* 97C30, 97C70.

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[†]S. Dehaene, *The Number Sense: How the Mind Creates Mathematics*. Oxford University Press, 2000. ISBN-10: 0195132408, ISBN-13: 978-0195132403.

[‡]I. V. Arnold, *Principles of selection and composition of arithmetic problems*, *Izvestia APN RSFSR* 1946, vyp. 6, 8–28. (Russian.)

[§]Khan Academy. <http://www.khanacademy.org/about>. Last Accessed 14 Apr 2011.

A piecewise course is open to piecewise assessment and analysis.

2. *A word problem from the Khan Academy*

In 2011 I was asked by my American colleagues to give my assessment of mathematical material on the Khan Academy website. Among other things I looked for the so-called “word problems” and clicked on a link leading to what was called there an “average word problem” but happened to be a “word problem about averages”.

Gulnar has an average score of 87 after 6 tests. What does Gulnar need to get on the next test to finish with an average of 78 on all 7 tests?

3. *Hints given by the Khan Academy*

What follows are hints provided, one after another, by the Khan Academy website:

Hint 1. Since the average score of the first 6 tests is 87, the sum of the scores of the first 6 tests is $6 \times 87 = 522$.

Hint 2. If Gulnar gets a score of x on the 7th test, then the average score on all 7 tests will be:

$$\frac{522 + x}{7}.$$

Hint 3. This average needs to be equal to 78 so:

$$\frac{522 + x}{7} = 78.$$

Hint 4 $x = 24$.

4. *“Questions” method for word problems*

And here is the same problem solved by the “steps” or “questions” method.

Question 1. How many points in total did Gulnar get in 6 tests?

Answer: $6 \times 87 = 522$.

Question 2. How many points in total does Gulnar need to get in 7 tests?

Answer: $7 \times 78 = 546$.

Question 3. How many points does Gulnar need to get in the 7th test?

Answer: $546 - 522 = 24$.

5. *Self-directing questions*

Crucially, the whole point of the “questions” method is that questions are not supposed to be asked by a teacher: students are taught to formulate these questions *themselves*.

Teaching the “questions method” focuses on the development of each student’s

ability to start his/her “questions method” attempt at a word problem asking *himself or herself* appropriate *self-directing questions* (they are called *auxiliary questions* in the Russian pedagogical literature, but in this country, the words “an auxiliary question” are loaded with expectation that the question is asked by a teacher to help a struggling pupil).

In the case of Gulnar’s problem, these self-directing questions are likely to be something like

Question A. “Gulnar has an average score of 87 after 6 tests.” *What questions can be asked about these data?*

Question B. “Gulnar needs to get an average of 78 on all 7 tests”. *What questions can be asked about these data?*

Therefore the use of the “questions” method in mathematics education involves gently nudging a child towards *reflection* and analysis of his / her own thought process. This should be done, it needs to be emphasised, at a level actually accessible to the child—and this can be done, as it was confirmed by mathematics education practice of dozens of countries around the world. I prefer the term “questions method” to the more commonly used, in British education literature, name “steps method” because the word “questions” emphasises the pro-active and reflective components of thinking, while the word “steps” might inadvertently imply a passive procedural approach.

As a child, I learnt the “questions” method in my primary school in a direct face-to-face communication with a live teacher and with my peers, and I describe it here as it was widely and routinely used in all primary schools in Russia in the 1960s. A colleague, responding to an earlier version of my notes on the “questions method”, indicated that I was lucky to have an “excellent mentor” who was using “the richness of the Socratic questioning”. I loved my teacher—but it needs to be explained that she was a village school teacher in Siberia and was educated (up to the age of 16) in the same village school and then for two years (up to the age of 18) in a pedagogical college in the town of Kyakhta—look it up on the GOOGLE map! Places more in the middle of nowhere can be found only on the Central Antarctic Plate.

If “policymakers” will ever read my paper, this is my message to them:

My teacher’s skills in arithmetic were a guaranteed and enforced minimum compulsory for every teacher.

6. *A comparison between the two approaches*

However, it is instructive to compare the two approaches in more detail.

The Khan Academy’s solution is longer than the “questions method” solution because Hints 2 and 3 are already equivalent to a solution of a 3-step problem (you need to express algebraically the new total, compute the new mean and compare with the desired result).

The “step” or “questions” method of solving the problem directly encourages students to look at the core of the concept of average, it invites them to re-visit

previous material and solve the problem of finding the total from the average. It encourages them to *talk* about mathematics, with themselves and with others.

In more technical terms, the “questions” method triggers the all-important dynamics of encapsulation / de-encapsulation of the concept of “average” (and we have to remember that this is a pretty abstract concept—this is understood even in the mass culture, in expressions such as *2point4 children*). The terms “encapsulation” and “de-encapsulation” are less frequently used, and perhaps a few words of clarification may be useful; I quote Weller et al.[†]:

The encapsulation and de-encapsulation of processes in order to perform actions is a common experience in mathematical thinking. For example, one might wish to add two functions f and g to obtain a new function $f + g$. Thinking about doing this requires that the two original functions and the resulting function are conceived as objects. The transformation is imagined by de-encapsulating back to the two underlying processes and coordinating them by thinking about all of the elements x of the domain and all of the individual transformations $f(x)$ and $g(x)$ at one time so as to obtain, by adding, the new process, which consists of transforming each x to $f(x)+g(x)$. This new process is then encapsulated to obtain the new function $f+g$.

Mathematical concepts are shaped and developed in a child’s mind in a recurrent process of encapsulation and de-encapsulation. Returning to solution of Gulnar’s problem by “questions” method, we see this process in action.

And what is even more important, self-directing questions are *meta-questions*, that is, questions aimed at finding the optimal way of reasoning.

From a pretty basic pedagogical point of view, if the didactic aim of the problem is to reinforce the understanding of averages, then the “questions” method appears to be more useful; it gives a student a joint and cohesive vision of the concept.

But I claim much more: the “questions” method can act as a trigger of the fundamental recurrent cycle of mathematical thinking: *intentional and purposeful* encapsulation / de-encapsulation of abstract entities. Like breathing and heartbeat in a living body, the encapsulation / de-encapsulation cycle makes the difference between live mathematics and zombie mathematics.

7. *Some further discussion: Charles Bingham on Khan Academy and “Mute Teaching”*

The inadvertent appearance of the trully archaic word “Socratic” in the comparison of the “questions method” approach and the Khan Academy’s approach to Gulnar’s problem reflects the deep socio-economic difference between the two models of mass education: the old one based on training and employment of an army of teachers, and the new one rooted in the emerging distributed cloud-based tech-

[†]K. Weller, A. Brown, E. Dubinsky, M. McDonald and C. Stenger, Intimations of infinity, Notices AMS 51 no. 7 (2004) 741–750.

nology of knowledge delivery and, in addition, affected by yet another (and still under-recognised) tectonic shift in the information technology:

“*Software is the infrastructure, and hardware the consumable.*”[†]

For that reason it could be illuminating to continue discussion with a few quotes from Charles Bingham’s paper with the bold title *Two educational ideas for 2011 and beyond* [[‡]].

In a nutshell, Bingham characterises the Khan Academy’s approach to education as *mute teaching*. In this paper, I used a simple example from mathematics materials of the Khan Academy to illustrate that the “Mute Teacher” model of mathematics education enforces a different—and, in my opinion, inferior—didactics of mathematics teaching in comparison with the “Teacher as Relator”. But Bingham does not make judgement, he simply describes what he sees:

... in 2011, if one looks outside of educational theory and educational practice, something else is also happening. It is happening without any heed being paid to the rhetoric of intensified progressivism, and without heed to the figure of the knowledgeable speaker. It is this something else that I want to emphasize. This something else might be called *The Event of the Mute Teacher*.

The “Event of the Mute Teacher” is not a surprise for anyone who teaches mathematics in a British university: we have already witnessed the “Event of the Mute Student”; in our students, the ability to communicate mathematics is suppressed by their pre-conditioning in the secondary school examination system entirely based on written examinations and questions cut into bite-sized chunks. However, Bingham points to a new aspect of this phenomenon: the impact of technological changes.

I offer this one simple example. On the web page entitled “Khan Academy,” one finds the following message: “Watch. Practice. Learn almost anything—for free.” And under the Khan Academy logo, 44,942,989 lessons delivered is proudly displayed (Khan 2011a)[§]. Also on the web page is a link to a video wherein Bill Gates touts the importance of Khan Academy.

Bill Gates’ words, as quoted by Bingham, deserve some special attention:

“What Sal Khan has done,”
notes Gates,

“is amazing He’s taken all this material and broken them (sic) down into little 12 minute lectures. I use it myself to remind myself of things. Children like it. So I was super happy when he came up and we got to talk about—Where does he go now? How can my foundation help him pursue this dream, and connect it up with the other great things going on on the internet? . . . So I see Sal Kahn as a pioneer in an

[†]From the submission by the Software Sustainability Institute to the House of Lords Select Committee on Science and Technology, 2nd Report of Session 2013–14 *Scientific Infrastructure*, <http://www.publications.parliament.uk/pa/ld201314/ldselect/ldsctech/76/76.pdf>. See also <http://www.software.ac.uk/news/2013-11-22-investing-uk-e-infrastructure-not-just-machines>, accessed 31 December 2013.

[‡]C. Bingham, *Two educational ideas for 2011 and beyond*, Stud. Philos. Educ. (2011). DOI 10.1007/s11217-011-9253-8.

[§]Khan Academy. <http://www.khanacademy.org/about>. Last Accessed 14 Apr 2011.

overall movement to use technology to let more and more people learn things, know where they stand. It's the start of a revolution ¶.

And Bingham continues his description of the Khan Academy:

Khan Academy consists of thousands of video clips that enable one to teach oneself whatever one wants. And as the website boasts, these videos are free, they are of high quality, and they can certainly do what they say they can do. They enable anyone to learn anything. Admittedly, the Khan Academy is more heavily focused on math and science. Admittedly, too, the academy, with its sequential staging of online pedagogy, has a sort of pre-modern, hyperrealist understanding that knowledge is something 'out there' written on some cosmic tablet. But it is not at all my aim to critique the web site, nor its epistemological assumptions. My point is rather to offer the Khan Academy as one example of thousands of other such places where one can become educated other than in a school or a formal educational institution.

Finally, this is how Bingham's views the impact of technological changes on the role of teacher in education:

Now, if we layer these sorts of pedagogical spaces onto the figure of the speaking teacher who finds it necessary to communicate ideas and to be well understood, we find that the well-spoken teacher is not really necessary in this year of 2011. What I mean by this is that there are thousands of speaking teachers available at anyone's fingertips, and it is very likely that the teacher one encounters at school or at university is less knowledgeable and less clear-spoken than another teacher who can be found online. For each teacher, there are quite a few Khan Academy's. It is in this sense that I say that the event of the mute teacher is upon us. There is not much need any more for the teacher to speak clearly. As Khan Academy boasts,

"Views: 24,000,000. Faculty: 1" †.

Others are always at hand to speak for him or her, and to speak better.

I have to admit that I disagree with Bingham, and I will try to explain my position in future papers, again using word problems as simple and efficient litmus test.

Disclaimer

The author writes in his personal capacity and the views expressed do not necessarily represent position of his employer or any other organisation or institution.

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¶Khan Academy. <http://www.khanacademy.org/about>. Last Accessed 14 Apr 2011.

† *Ibid.*