Selected Passages from Correspondence with Friends 1 no. 1 (2013), 1–2

A MONOTONOUS BOUNDED SEQUENCE AS A METAPHOR FOR MATHEMATICS EDUCATION

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To the best of my knowledge, the vast majority of mathematics departments in this country have already succumbed to the pressure to slice examination questions into sub-questions and guide the candidate through an increasingly long sequence of preset steps and sub-steps.

And here is an example taken from a website of one of the better known universities; it appears to be a good simile for effects of this examination setup on the quality of education.

1. (a) [7 marks] Show that if $\{a_n\}$ is a bounded monotone sequence of real numbers then it is convergent.
   (b) [13 marks] The sequence $\{a_n\}$ is defined by

$$a_0 = c, \quad (\alpha+\beta)a_{n+1} = (a_n)^2 + \alpha\beta;$$

   where $0 < \alpha < c < \beta$.
   (i) Prove that, if $\{a_n\}$ converges to a limit a, then $a = \alpha$ or $a = \beta$.
   (ii) Prove that $a_{n+1} - \alpha$ and $a_n - \alpha$ have the same sign, and also $a_{n+1} - \beta$ and $a_n - \beta$ have the same sign.
   (iii) Prove that $a_{n+1} < a_n$ and hence that $a_n \to \alpha$.

I argue that the label 1(b)(iii) and the word “hence” in this problem mark the beginning of the end of the candidate’s mathematics education.

To illustrate that, let me re-formulate the problem in a less fractured way.

1. Show that if $\{a_n\}$ is a bounded monotone sequence of real numbers then it is convergent. Then use this fact to show that if the sequence $\{a_n\}$ is defined by

$$a_0 = c, \quad (\alpha+\beta)a_{n+1} = (a_n)^2 + \alpha\beta;$$

   where $0 < \alpha < c < \beta$, then

$$a_n \to \alpha.$$

2010 *Mathematics Subject Classification* 97D60, 97I30.

Of course, the second version is harder—but it is significantly less confusing for a stronger student than the messed-up original version. I also believe that students able to cope with the second version have a good chance to progress further in their study of mathematics. On the contrary, I think that slicing the question into steps is a tacit admission, on part of the examiners, that their students have reached their limits and are unlikely to progress further. Following hints is not mathematics problem-solving!

I believe that assessment based on the assumption that all students' intellectual potential is limited is unfair not only to stronger students, it is unfair to *all* students. I adhere to John Dewey's maxim:

> *The aim of education is to enable individuals to continue their education ... [and] the object and reward of learning is continued capacity for growth. Now this idea cannot be applied to all the members of a society except where intercourse of man with man is mutual, and except where there is adequate provision for the reconstruction of social habits and institutions by means of wide stimulation arising from equitably distributed interests. And this means a democratic society.*

Unfortunately, as I see it, examination practices of British universities limit the prospects of our students' growth as mathematicians.

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